

Lesson 68

Introduction to Trigonometric Ratios

Trigonometry is the study of the relationship between sides and angles of triangles. There are three basic ratios in trigonometry that can be used to find measures in right triangles.

The three ratios are the sine of an angle, the cosine of an angle, and the tangent of an angle. A trigonometric ratio is a ratio of two sides of a right triangle.

Trigonometric Ratios

In a right triangle, the sine of an angle is the ratio of the length of the leg opposite the angle to the length of the hypotenuse.

$$\sin A = \frac{\textit{Opposite}}{\textit{Hypotenuse}}$$

In a right triangle, the cosine of a triangle is the ratio of the length of the leg adjacent to the angle to the length of the hypotenuse.

$$\cos A = \frac{\textit{Adjacent}}{\textit{Hypotenuse}}$$

In a right triangle, the tangent of an angle is the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle.

$$\tan A = \frac{\textit{Opposite}}{\textit{Adjacent}}$$

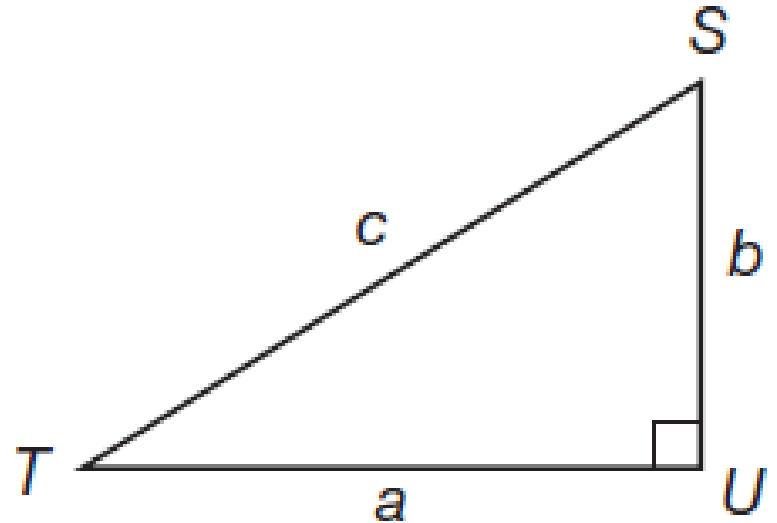
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For example:

The sine of $\angle T$ in ΔSTU is $\frac{b}{c}$.

The cosine of $\angle T$ is $\frac{a}{c}$.

The tangent of $\angle T$ is $\frac{b}{a}$.



Example 1 Calculating Trigonometric Ratios

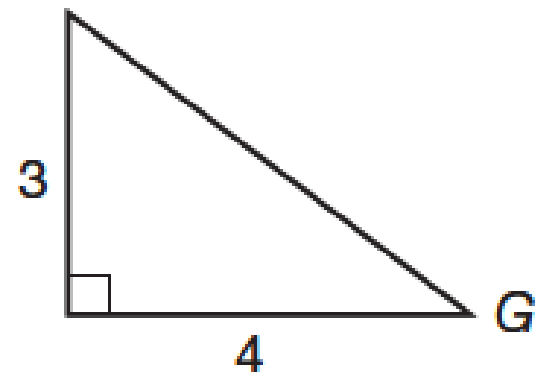
a. Give the sine, cosine, and tangent of $\angle G$.

SOLUTION

Find the hypotenuse of the triangle using the Pythagorean Theorem.

The hypotenuse is 5.

$$\sin G = \frac{3}{5}$$
$$\cos G = \frac{4}{5}$$
$$\tan G = \frac{3}{4}$$



A calculator can be used to evaluate the cosine, sine, and tangent of an angle.

Example 2 Calculating Trigonometric Ratios

Use a calculator to evaluate each expression. Round the answer to the nearest hundredth.

a. $\cos 72^\circ$

SOLUTION

$$\cos 72^\circ = 0.31$$

b. $\sin 30^\circ$

SOLUTION

$$\sin 30^\circ = 0.5$$

c. $\tan 70^\circ$

SOLUTION

$$\tan 70^\circ = 2.75$$

Trigonometric ratios can be used to solve for unknown side lengths in right triangles. An equation can be divided or multiplied by a trigonometric ratio, just as it can with any real number.

Example 3 Solving for Side Lengths Using Trigonometry

Use the tangent ratio to find e to the nearest hundredth.

SOLUTION

$$\tan 72^\circ = \frac{e}{13}$$

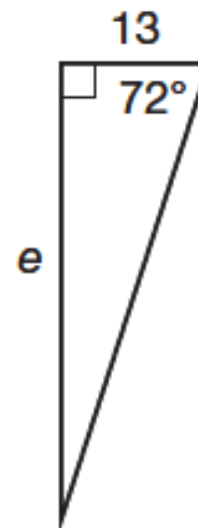
$$13 \tan 72^\circ = e$$

$$e \approx 40.01$$

Tangent function

Multiply both sides by 13.

Simplify.



Example 4 More Solving for Side Lengths

Use the sine ratio to find x to the nearest hundredth.

SOLUTION

$$\sin 32^\circ = \frac{8}{x}$$

$$x \sin 32^\circ = 8$$

$$x = \frac{8}{\sin 32^\circ}$$

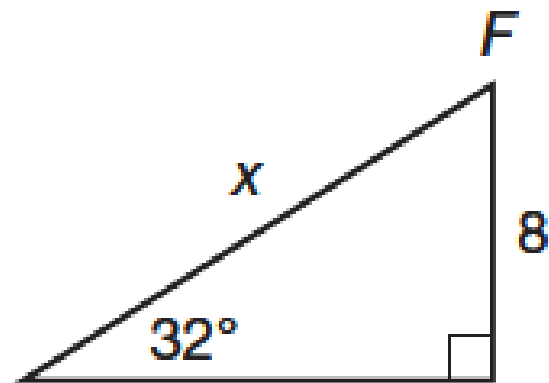
$$x \approx 15.10$$

Sine function

Multiply both sides by x .

Divide both sides by $\sin 32^\circ$.

Simplify.



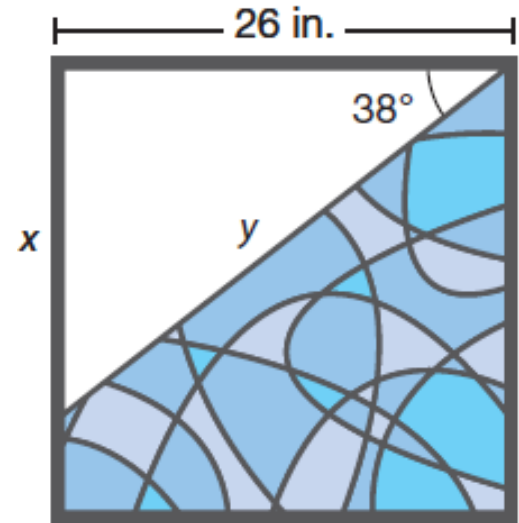
Example 5 Application: Art

Artists who make stained glass windows use right triangles in their patterns. If an artist is making a stained glass window for a square window with sides that are 26 inches long, what is the value of x and y in the diagram? Give answers to the nearest hundredth.

SOLUTION

Use trigonometric ratios to solve for x and y .

$$\begin{aligned}\tan 38^\circ &= \frac{x}{26} \\ 26 \tan 38^\circ &= x \\ x &\approx 20.31\end{aligned}$$
$$\begin{aligned}\cos 38^\circ &= \frac{26}{y} \\ y &= \frac{26}{\cos 38^\circ} \\ y &\approx 33.00\end{aligned}$$



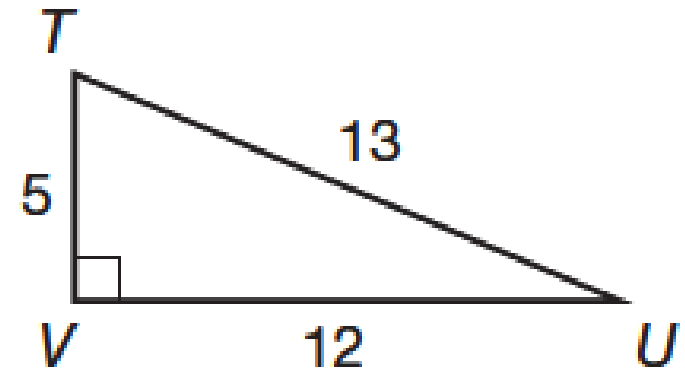
The other leg of the right triangle measures approximately 20.31 inches, and the hypotenuse is approximately 33 inches.

You Try!!!!

Use the figure to answer problems a and b.

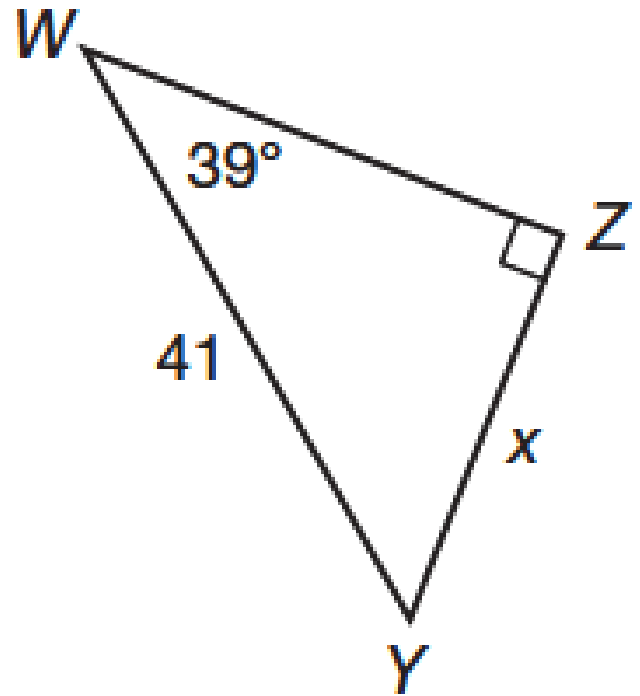
a. What is the sine of $\angle T$?

b. What is the tangent of $\angle U$?



You Try!!!!

c. Find x to the nearest hundredth.



You Try!!!!

Evaluate each expression.

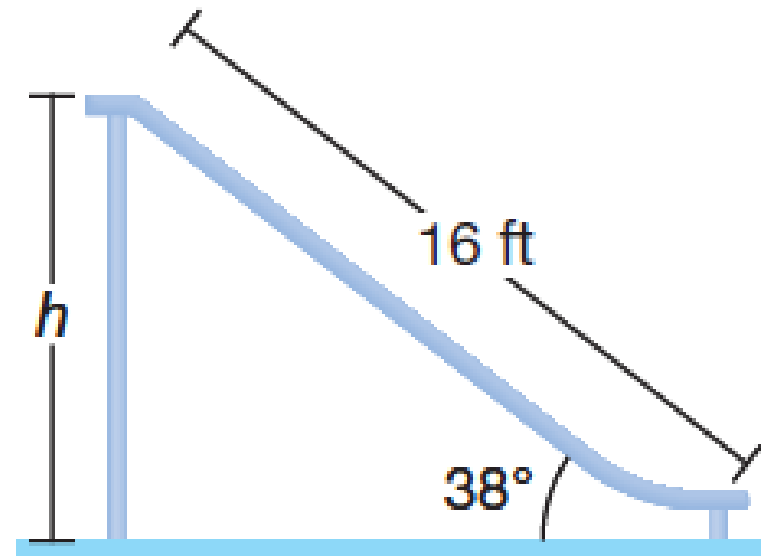
$$d.\sin 30^\circ$$

$$e.\cos 90^\circ$$

$$f.\tan 45^\circ$$

You Try!!!!

g. A playground has a slide that is at a 38° angle with the ground. If the slide is 16 feet long, what is the height?



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)