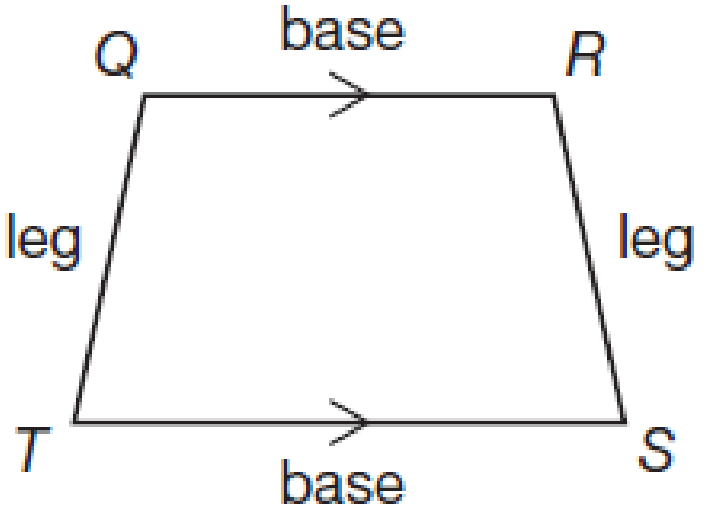
Lesson 69

Properties of Trapezoids and Kites

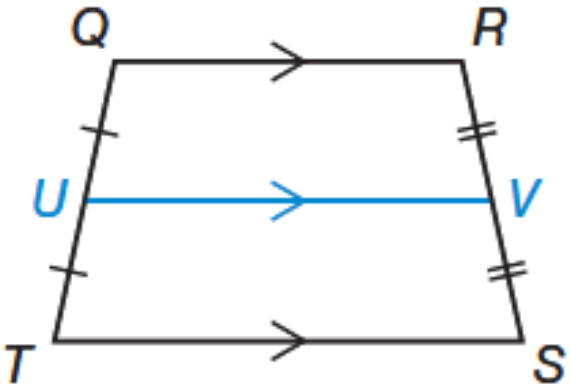
The bases of a trapezoid are its two parallel sides. A base angle of a trapezoid is one of a pair of consecutive angles whose common side is a base of the trapezoid. Trapezoids have two pairs of base angles. The legs of a trapezoid are the two nonparallel sides.

Figure $QRST$ is a trapezoid.
$\overline{QR}$ and $\overline{TS}$ are bases,
$\angle Q$ and $\angle R$ are base angles,
$\angle T$ and $\angle S$ are base angles,
And $\overline{QT}$ and $\overline{RS}$ are legs of the trapezoid.



The midsegment of a trapezoid – The segment whose endpoints are the midpoints of the legs of the trapezoid.

Theorem 69-1: Trapezoid Midsegment Theorem – The midsegment of a trapezoid is parallel to both bases and has a length that is equal to half the sum of the bases. Therefore, if $\overline{UV}$ is the midsegment of trapezoid *QRST*, then $\overline{UV} \parallel \overline{QR}$, $\overline{UV} \parallel \overline{TS}$, and $UV = \frac{1}{2}(QR + TS)$.



Example 1 Applying Properties of the Midsegment of a Trapezoid

The midsegment of trapezoid *ABCD* is $\overline{EF}$. Find the length of $\overline{EF}$.

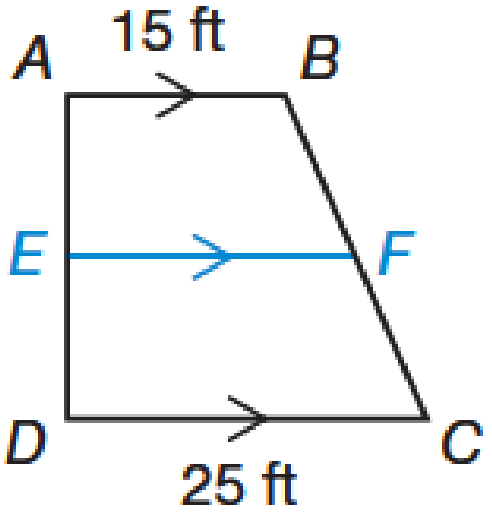
SOLUTION

$$EF = \frac{1}{2}(AB + DC)$$

$$EF = \frac{1}{2}(15 + 25)$$

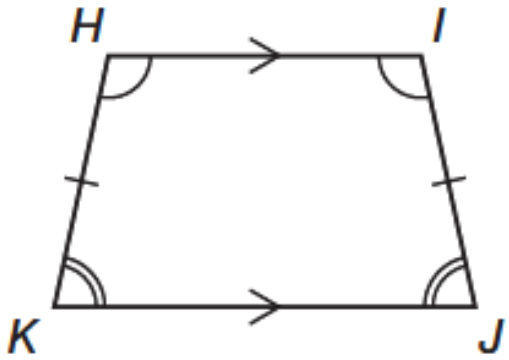
$$EF = 20$$

The length of $\overline{EF}$ is 20 feet.



An isosceles trapezoid is a trapezoid with congruent legs. Like isosceles triangles, isosceles trapezoids have congruent base angles.

Properties of Isosceles Trapezoids – Base angles of an isosceles trapezoid are congruent. If trapezoid *HIJK* is isosceles, then $\angle H \cong \angle I$ and $\angle J \cong \angle K$.



Example 2 Applying Properties of the Base Angles of an Isosceles Trapezoid

Find the measures of $\angle N$, $\angle O$, and $\angle P$ in isosceles trapezoid $MNOP$.

SOLUTION

Because the trapezoid is isosceles, its base angles are congruent.

Therefore, $\angle M \cong \angle N$ and $\angle P \cong \angle O$.

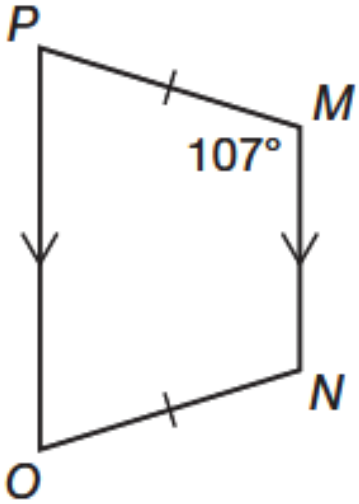
Therefore, $m\angle N = 107°$.

Notice that $\overline{MP}$ is a transversal that intersects two parallel lines. Therefore, $\angle M$ and $\angle P$ are supplementary.

$m\angle P = 180° - 107°$

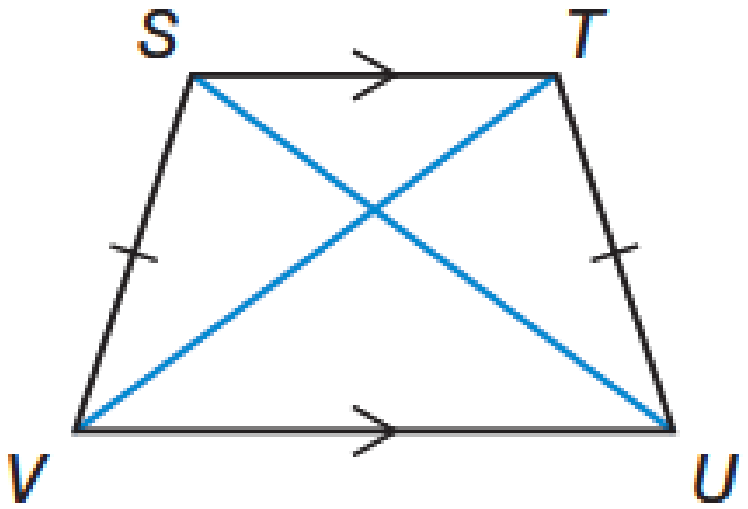
$m\angle P = 73°$

$m\angle O = 73°$



Properties of Isosceles Trapezoids – The diagonals of an isosceles trapezoid are congruent.

In isosceles trapezoid *STUV*, $\overline{SU} \cong \overline{TV}$.



Example 3 Applying Properties of the Diagonals of an Isosceles Trapezoid

ABCD is an isosceles trapezoid. Find the length of $\overline{CE}$ if $AC = 22.3$ centimeters and $AE = 8.9$ centimeters.

SOLUTION

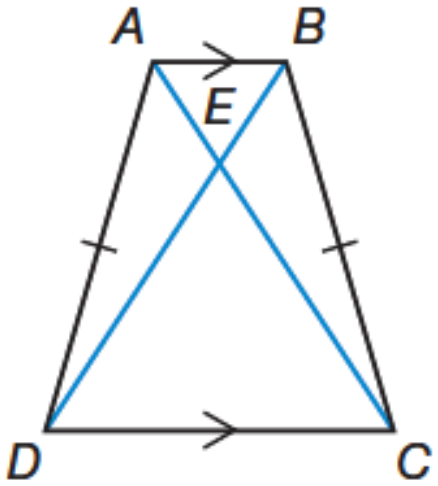
Because $\overline{AC}$ and $\overline{BD}$ are the diagonals of an isosceles trapezoid, they are congruent.

$CE = AC - AE$

$CE = 22.3 - 8.9$

$CE = 13.4$

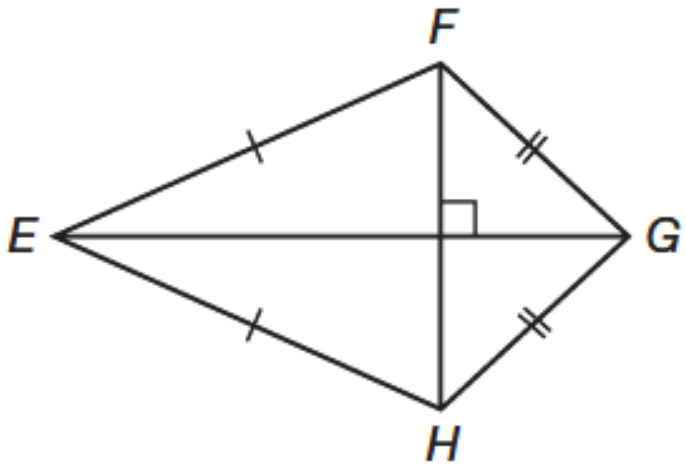
The length of $\overline{CE}$ is 13.4 centimeters.



Recall that kites are quadrilaterals with exactly two pairs of congruent adjacent sides.

Properties of Kites – The diagonals of a kite are perpendicular.

$$\overline{EG} \perp \overline{FH}$$



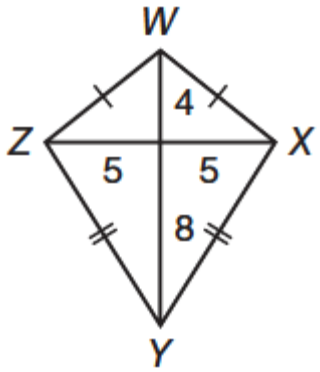
Example 4 Applying Properties of the Diagonals of a Kite

Find the lengths of the sides of kite *WXYZ*. Round to the nearest tenth.

SOLUTION

Because the diagonals of a kite are perpendicular to each other, the Pythagorean Theorem can be used to find the length of each side.

$$WX^2 = 4^2 + 5^2$$
$$WX^2 = 41$$
$$WX \approx 6.4$$



Since *WXYZ*, is a trapezoid, *WX* and *WZ* are congruent.

Therefore, *WZ* is also approximately 6.4.

$$YZ^2 = 8^2 + 5^2$$
$$YZ^2 = 89$$
$$YZ \approx 9.4$$

$\overline{YZ}$ and $\overline{YX}$ are also congruent, so *YX* is approximately 9.4.

Example 5 Application: Woodworking

A carpenter is making an end table with a trapezoid-shaped top. There will be three glass panels on the top of the table, as shown in the diagram. In the trapezoid *BDEG*, $\overline{CF}$ is a midsegment. In the trapezoid *ACFH*, $\overline{BG}$ is a midsegment. What are the lengths of $\overline{CF}$ and $\overline{DE}$?

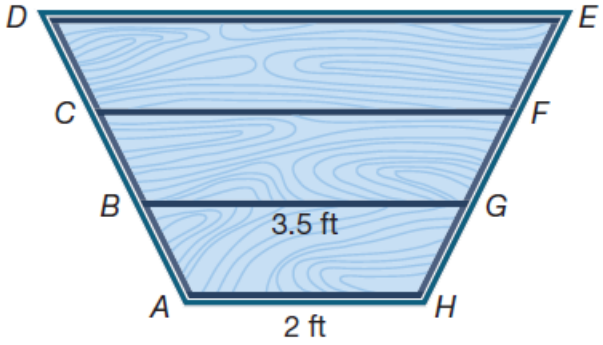
SOLUTION

Since $\overline{BG}$ is a midsegment of *ACFH*, its length is half the sum of *CF* and *AH*.

$BG = \frac{1}{2}(AH + CF)$	Midsegment of a trapezoid
$3.5 = \frac{1}{2}(2 + CF)$	Substitute.
$CF = 5$ feet	Solve.

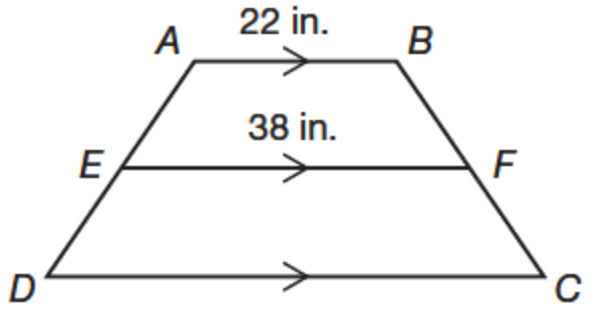
$\overline{CF}$ is the midsegment of *BDEG*, so:

$CF = \frac{1}{2}(DE + BG)$	Midsegment of a trapezoid
$5 = \frac{1}{2}(DE + 3.5)$	Substitute.
$DE = 6.5$ feet	Solve.

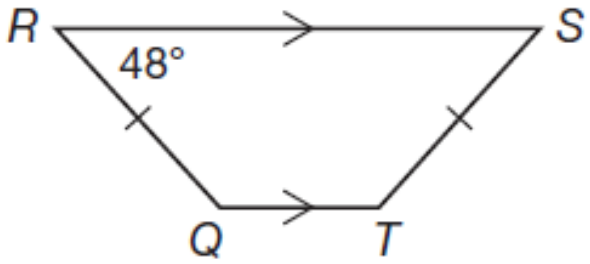


You Try!!!!

a. In the diagram, $\overline{EF}$ is the midsegment of trapezoid *ABCD*. Find the length of $\overline{CD}$.

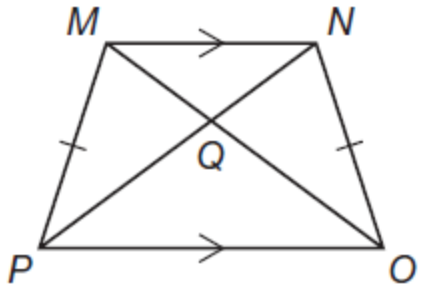


b.Find the measures of $\angle Q$, $\angle S$, and $\angle T$ in trapezoid *QRST*.

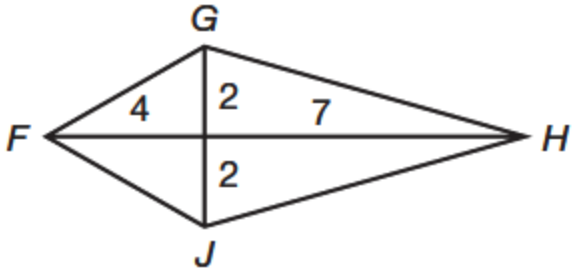


You Try!!!!

c.In isosceles trapezoid *MNOP*, find the length of $\overline{MQ}$ if *NP* = 17.5 yards and *PQ* = 9.6 yards.

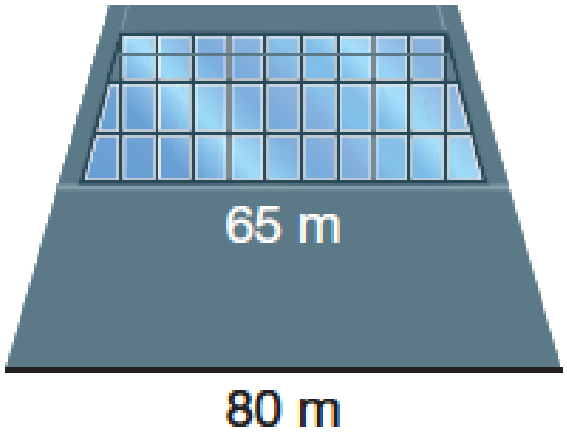


d.Find the lengths of the sides of kite *FGHJ*. Round the lengths to the nearest tenth.



You Try!!!!

e. The side of a building is shaped like a trapezoid. The base of a row of windows runs along the midsegment of this trapezoid. What is the length of the building's roof?



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1-30 (Do the starred ones first)