Lesson 70

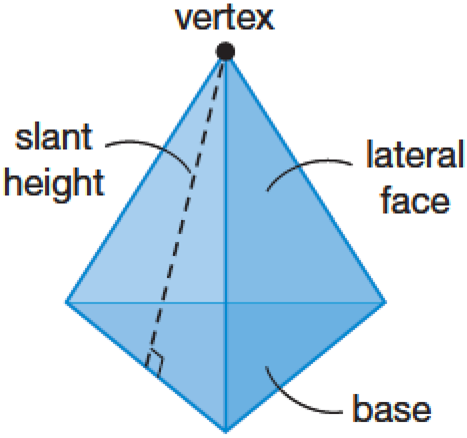
Finding Surface Areas and Volumes of Pyramids

The vertex of a pyramid is the common vertex of the pyramid's lateral faces.

The base of a pyramid is the face of the pyramid that is opposite the vertex.

A regular pyramid is a pyramid with a regular polygon as a base and with lateral faces that are congruent isosceles triangles.

The slant height of a regular pyramid is the distance from the vertex of a regular pyramid to the midpoint of an edge of the base.



Lateral Area Formula for Regular Pyramids – The lateral area, L, of a regular pyramid is given by the formula below, where P is the perimeter of the base and l is the slant height.

$$L = \frac{1}{2}Pl$$

Example 1 Calculating Lateral Area of a Pyramid

What is the lateral area of this regular pentagonal pyramid?

SOLUTION

Since the base is a regular pentagon with side lengths of 2 centimeters, its perimeter is 10 centimeters.

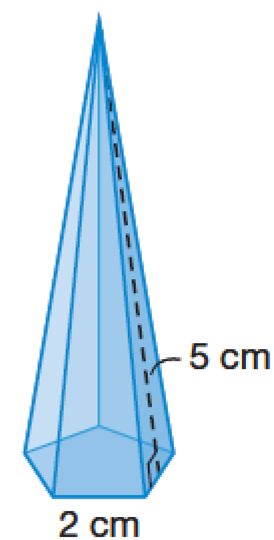
Substitute the perimeter and the slant height into the lateral area formula for regular pyramids.

$L = \frac{1}{2}Pl$ Lateral surface area for regular pyramids

$L = \frac{1}{2}(10)(5)$ Substitute.

$L = 25cm^2$ Simplify.

The lateral area is 25 square centimeters.



The sum of the base area and the lateral area is the total surface area of the pyramid.

Surface Area of a Pyramid – The total surface area, S, of a pyramid is given by the formula below, where L is the lateral surface area and B is the area of the base.

$$S = L + B$$

Example 2 Calculating Surface Area of a Pyramid

Calculate the total surface area of a regular hexagonal pyramid with a slant height of 12 centimeters and a base side that is 3 centimeters long.

SOLUTION

Since the base is a regular hexagon with 3-centimeter side lengths, its perimeter is 18 centimeters.

Use the formula for the area of a regular hexagon that was discussed in Lesson 66, where *s* is the side length.

$$B = \frac{1}{2}\left(\frac{s\sqrt{3}}{2}\right)P$$

$$B = \frac{1}{2}\left(\frac{3\sqrt{3}}{2}\right)18$$

$$B \approx 23.38\ cm^2$$

Hint

The formula for area of a regular hexagon is:

$A = \frac{1}{2}aP, a = \frac{s\sqrt{3}}{2}$.

So, $A = \frac{1}{2}\left(\frac{s\sqrt{3}}{2}\right)P$.

Example 2 Calculating Surface Area of a Pyramid

Calculate the total surface area of a regular hexagonal pyramid with a slant height of 12 centimeters and a base side that is 3 centimeters long.

SOLUTION

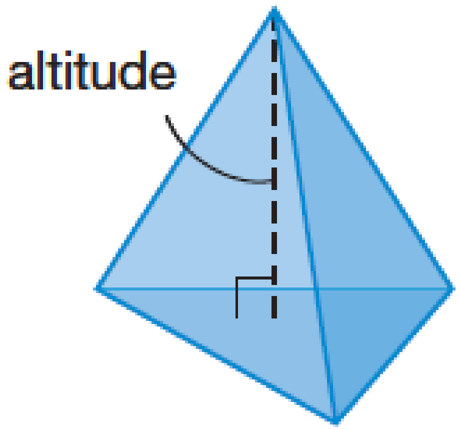
Now calculate the total surface area of the pyramid:

$S = L + B$	Surface area of a pyramid
$S = \frac{1}{2}Pl + B$	Substitute.
$S \approx \frac{1}{2}(18)(12) + 23.38$	Substitute.
$S \approx 131.38\ cm^2$	Simplify.

Therefore, the surface area of the pyramid is approximately 131.38 square centimeters.

The altitude of a pyramid is the perpendicular segment from the vertex to the plane containing the base. The length of the altitude is the height of the pyramid.

The volume of a pyramid can be found using the height and the area of the base.



Volume of a Pyramid – The volume, V, of a pyramid is given by the formula below, where B is the area of the base and h is the height.

$$V = \frac{1}{3}Bh$$

Recall that the volume of a prism is given by $V = Bh$. The volume of a pyramid is one third the volume of a prism with equal height and a congruent base.

Example 3 Calculating Volume of a Pyramid

Find the volume of the pyramid. The height is 9 inches and the base is a right triangle with legs that are 5 inches and 8 inches long, respectively.

SOLUTION

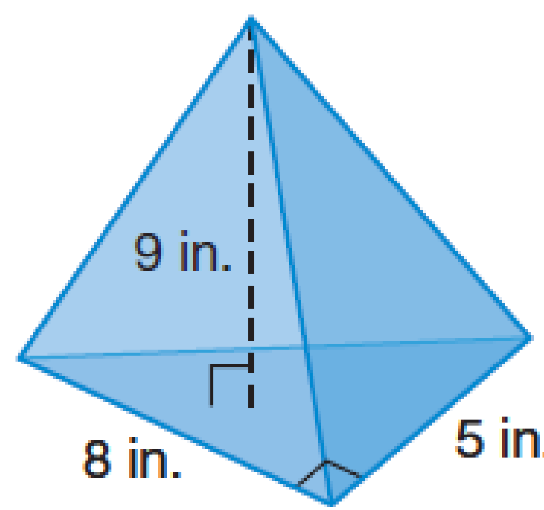
First find *B*, the base area.

$$B = \frac{1}{2}bh$$
$$B = \frac{1}{2}(8\ in.)(5\ in.)$$
$$B = 20in^2$$

Then find the volume, *V*.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(20in^2)(9in)$$
$$V = 60in^3$$

The volume of the pyramid is 60 cubic inches.



Example 4 Application: The Louvre Pyramid

The Louvre Pyramid, a regular square pyramid, is the main entrance of the Musée du Louvre in Paris. It has an approximate height of 70 feet and its square base has sides that are 115 feet long. What is the lateral area of the pyramid?

Example 4 Application: The Louvre Pyramid

SOLUTION

Since the base has four congruent sides of 115 feet each, the perimeter of the base is 460 feet.

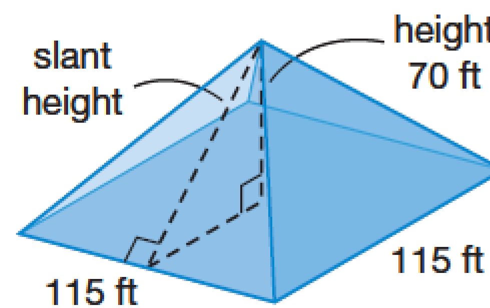
To find the slant height, use the Pythagorean Theorem.

One leg is the height, the other is the apothem of a square with a side length of 115 feet, which is simply half the length of the side.

$$l = \sqrt{70^2 + \left(\frac{115}{2}\right)^2}$$

$$l = \sqrt{4900 + \frac{13225}{4}}$$

$$l \approx 90.58\,ft$$



Now substitute into the formula for lateral surface area.

Example 4 Application: The Louvre Pyramid

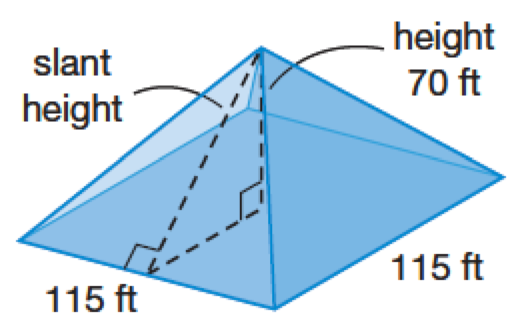
SOLUTION

$$L = \frac{1}{2}Pl$$

$$L \approx \frac{1}{2}(460)(90.58)$$

$$L \approx 20883.4\ ft^2$$

Thus, the lateral area of the Louvre Pyramid is approximately 20,833 square feet.



You Try!!!!

a.What is the lateral area of a regular octagonal pyramid with a side length of 5 centimeters and a slant height of 7 centimeters?

b. What is the surface area of a regular hexagonal pyramid with a slant height of 8 inches and a base side length of 4 inches, to the nearest hundredth of a square inch?

You Try!!!!

c.What is the volume of a square pyramid with side lengths of 5 feet and a height of 10 feet, to the nearest tenth of a square foot?

d.The Pyramid Arena in Memphis, Tennessee, is the third-largest square pyramid in the world. It is approximately 321 feet tall and the length of one side of the base is about 600 feet. What is its surface area?

Assignment

Page 467

Lesson Practice (Ask Mr. Heintz)

Page 467

Practice 1–30 (Do the starred ones first)