Lesson 71

Translations

A translation shifts every point of a figure the same distance in the same direction. A figure that is transformed by a translation remains congruent to its preimage. Its side lengths, angle measures, and other properties remain the same. Translation changes nothing but the location of a figure.

Translation – A translation is an isometry, meaning the preimage and its translated image are the same shape and size.

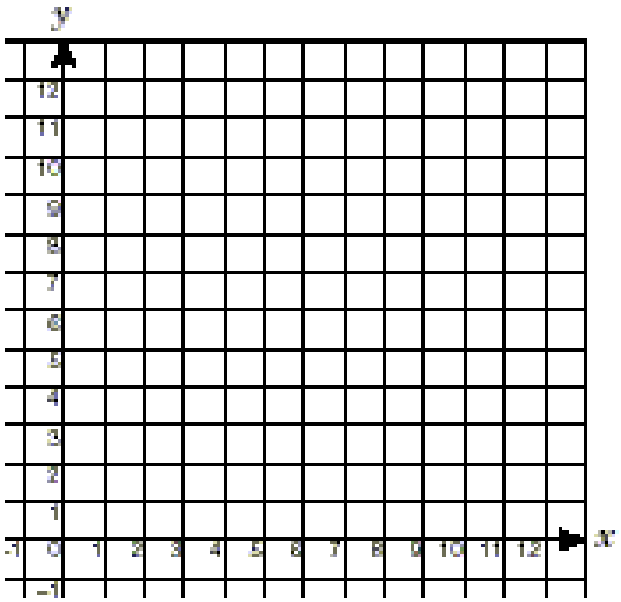
Example 1 Translations in One Dimension

A square has vertices $A(1, 1)$, $B(4, 1)$, $C(4, 4)$, and $D(1, 4)$.

a. Find the coordinates of the vertices of the image of square $ABCD$ after a translation of 5 units to the right. Show the preimage and image on the same coordinate grid.

SOLUTION

The x-coordinates of the vertices of the image of the square after it is translated 5 units to the right are 5 greater than the x-coordinates of the vertices of the preimage. The y-coordinates are unchanged.



Example 1 Translations in One Dimension

A square has vertices $A(1, 1)$, $B(4, 1)$, $C(4, 4)$, and $D(1, 4)$.

b. Show that the area of $A'B'C'D'$ is equal to the area of $ABCD$.

SOLUTION

In $ABCD$, $AB = 3$ and $BC = 3$, so the area of the square is 9 centimeters squared.

In $A'B'C'D'$, $A'B' = 3$ and $B'C' = 3$, so the area of the square is also 9 centimeters squared.

Mapping notation is used to indicate the way in which a point or several points should be transformed. An example of translation mapping notation is given below.

$T: (x, y) \rightarrow (x + 1, y - 2)$

This mapping says that in a transformation, the original pair of coordinates, (x, y), will be changed into $(x + 1, y - 2)$. That is, the x–coordinate will increase by 1 and the y–coordinate will decrease by 2.

Example 2 Translations in Two Dimensions

The vertices of a triangle are $X(-2, 0)$, $Y(-2, -4)$ and $Z(1, -4)$. Find the image of ΔXYZ after the translation $T: (x, y) \rightarrow (x + 5, y + 4)$. Show the preimage and image on the same coordinate grid.

SOLUTION

Graph the preimage triangle.

The translation moves every point 5 units to the right and 4 units up.

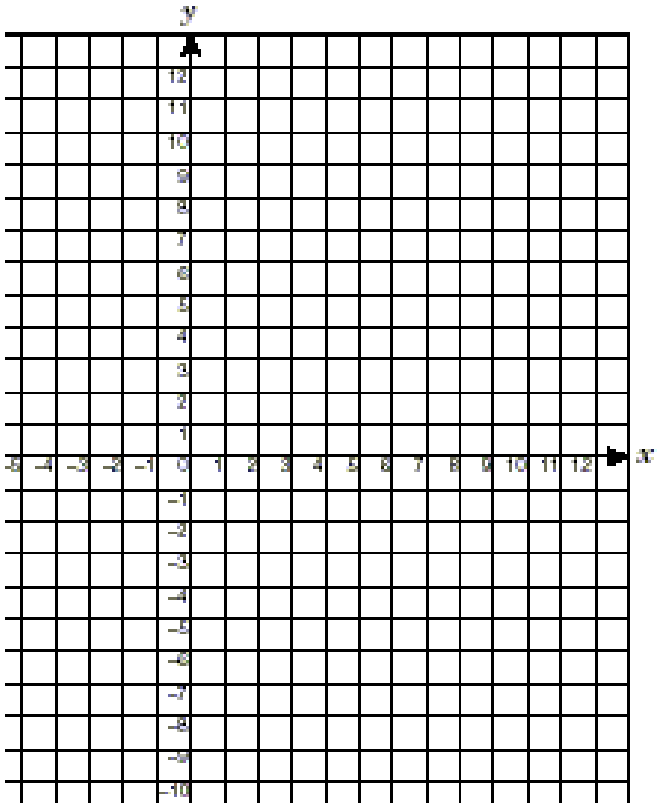
The transformation mapping is shown below.

$X(-2, 0) \rightarrow X'(3, 4)$

$Y(-2, -4) \rightarrow Y'(3, 0)$

$Z(1, -4) \rightarrow Z'(6, 0)$

Plot X', Y', and Z' and connect them to form $\Delta X'Y'Z'$.



A translation for a polygon can also be represented using a vector. Placing the initial point of the vector on each point of the preimage will indicate the position of the point in the image.

Example 3 Showing Translations with Vectors

Find the image of *ABCD* under the translation vector ⟨2, 3⟩.

SOLUTION

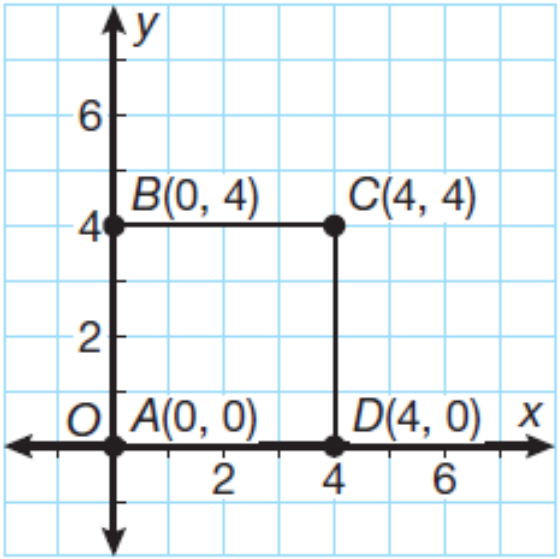
Place the initial point of the vector on point *A* and draw the vector, which moves each point 2 units right and 3 units up.

Drawing the vector shows that the new image point *A'* will be at (2, 3).

Repeat this process for points *B*, *C*, and *D*.

The image is shown in the diagram.

B' is at (2, 7), *C'* is at (6, 7), and *D'* is at (6, 3).



Example 4 Application: Computer Animation

A character in a new animated movie will move from the point (4, −5) first to the point (16, 3), then to point (13, −2). Find the vectors the animators need to apply to the character to make these two translations.

SOLUTION

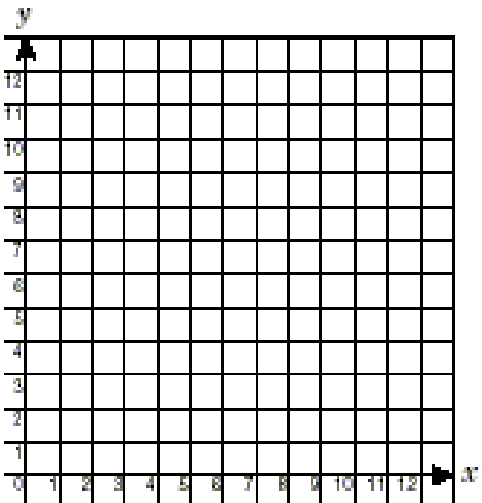
For the first translation, the change in x-values is 12, and the change in y-values is 8, so the first vector is $\langle 12, 8 \rangle$.

For the second translation, the change in x-values is −3, and the change in y-values is −5, so the second vector is $\langle -3, -5 \rangle$.

You Try!!!!

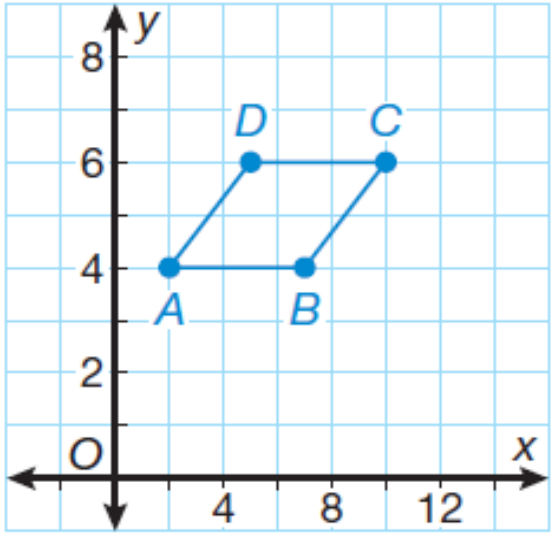
a.A line segment has endpoints $A(11, 5)$ and $B(4, 9)$. It is translated 5 units up. What are the coordinates of A' and B'?

b.A triangle has vertices $K(2, 5)$, $L(1, 11)$ and $M(5, 7)$. Find the image of ΔKLM after the translation. T: $(x, y) \rightarrow (x - 2, y + 1)$ Show the preimage and image on the same coordinate grid.



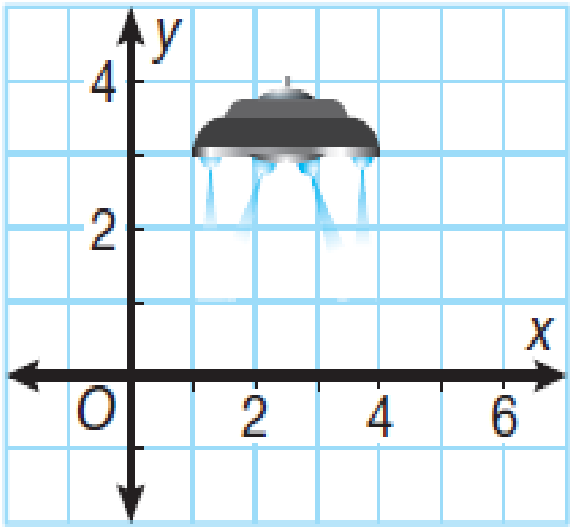
You Try!!!!

c.Find the coordinates of the vertices of the image of parallelogram *ABCD* after translation by the vector ⟨3, 1⟩.



You Try!!!!

d.In an animated cartoon, the UFO shown will move across the screen to the right. The animator translates two points of the UFO from (1, 3) and (4, 3) to an image at (9, 4) and (12, 4). Give the component form of the vector that describes this translation.



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1-30 (Do the starred ones first)