Lesson 72

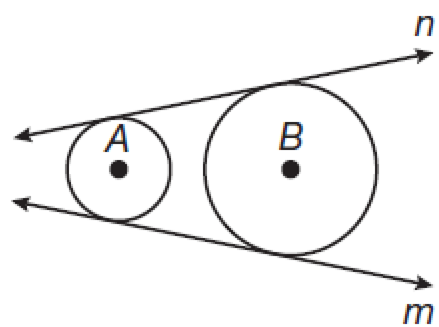
Tangents and Circles, Part 2

Tangent – A line that intersects a circle at exactly one point.

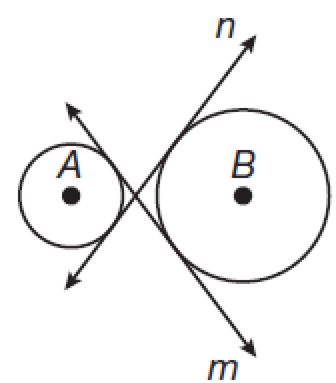
Point of Tangency – The intersection between the circle and the tangent.

Common Tangent – A tangent to two circles. Common tangents can be internal tangents or external tangents.

Recall Theorem 58-3: If two tangent segments are drawn to a circle from the same exterior point, then they are congruent.



External Common Tangents



Internal Common Tangents

Example 1 Solving Problems with Common Tangents

Given that $\overleftrightarrow{MR}$ and $\overleftrightarrow{PN}$ are internal common tangents to $\odot A$ and $\odot B$, find the length of $\overline{MQ}$.

SOLUTION

Since two segments tangent to a circle from the same exterior point are congruent, $\overline{NQ} \cong \overline{RQ}$ and $\overline{MQ} \cong \overline{PQ}$.

$NQ = RQ$ Definition of congruent segments

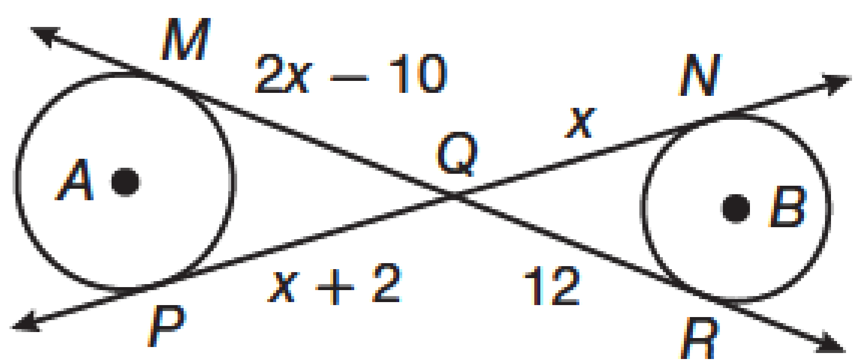
$x = 12$ Substitute

Substitute the value of x into the expression for the length of $\overline{MQ}$.

$MQ = 2x - 10$

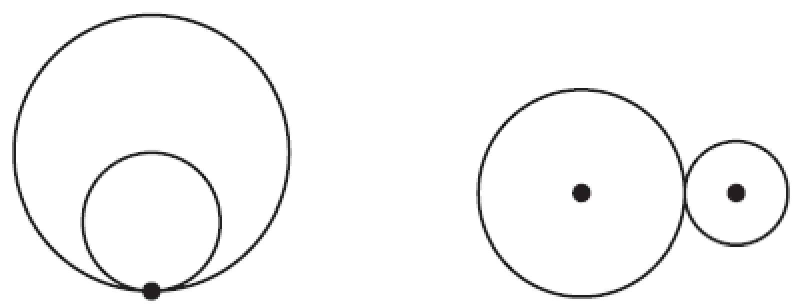
$MQ = 2(12) - 10$

$MQ = 14$



Tangent circles – coplanar circles that intersect at exactly one point.

Tangent circles can be internally tangent or externally tangent. In both cases, the radii of the two circles are collinear.



Internally Tangent Circles

Externally Tangent Circles

Example 2 Solving Problems with Tangent Circles

In the diagram, $\odot Q$ is tangent to $\odot M$ and $\overline{NP}$ is tangent to $\odot Q$. The radius of $\odot Q$ is 5 centimeters and the radius of $\odot M$ is 2 centimeters. Find the area of ΔQNP to the nearest square centimeter.

SOLUTION

Since the circles are tangent, they intersect at only one point, and their radii are collinear.

Since $\overline{QN}$ is composed of a radius of $\odot Q$ and a diameter of $\odot M$, its length is 9 centimeters.

$\overline{NP}$ is tangent to $\odot Q$, $m\angle QPN = 90°$.

So ΔQNP is a right triangle with a 9-centimeter hypotenuse and one 5-centimeter leg.

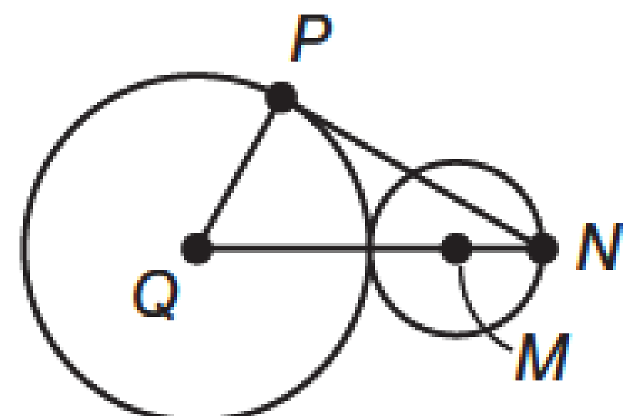
Use the Pythagorean Theorem to find the length of the other leg.

$QP^2 + PN^2 = QN^2$	Pythagorean Theorem
$5^2 + PN^2 = 9^2$	Substitute.
$PN = 2\sqrt{14}$	Solve.

Now the legs of the triangle can be used to find the area.

$A = \frac{1}{2}bh$	Area
$A = \frac{1}{2}(5)(2\sqrt{14})$	Substitute.
$A \approx 18.7$	Simplify.

So the area of the triangle is about 19 square centimeters.



Example 3 Application: Mechanics

A car has a timing belt that consists of two pulleys and a belt, as shown in the diagram. The belt runs around the two pulleys and is tangent to both of them. The dotted segments, $\overline{JI}$ and $\overline{JK}$, have been drawn into the diagram to assist in finding the distance between the two pulleys. Find *IH* and *KL*.

SOLUTION

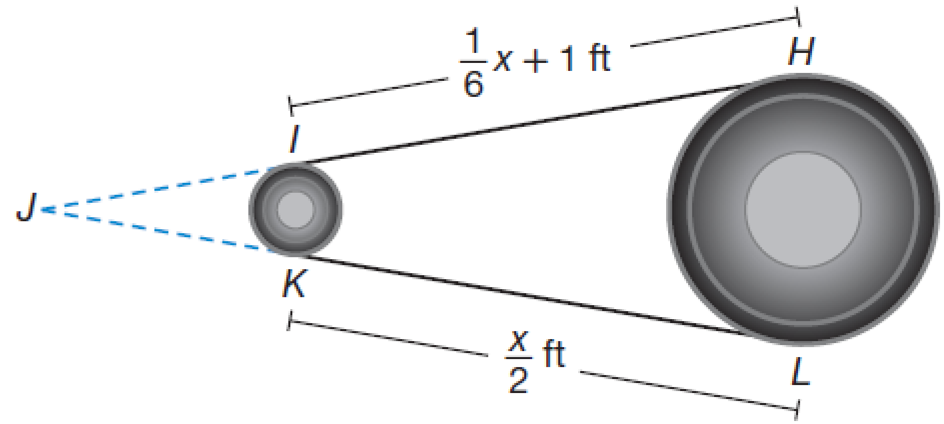
Since the pulley is tangent to the circle, and the tangent lines meet at point *J*, $\overline{JH} \cong \overline{JL}$ and $\overline{JI} \cong \overline{JK}$.

Therefore, $\overline{IH} \cong \overline{KL}$

$IH = KL$

$$\frac{1}{6}x + 1 = \frac{x}{2}$$

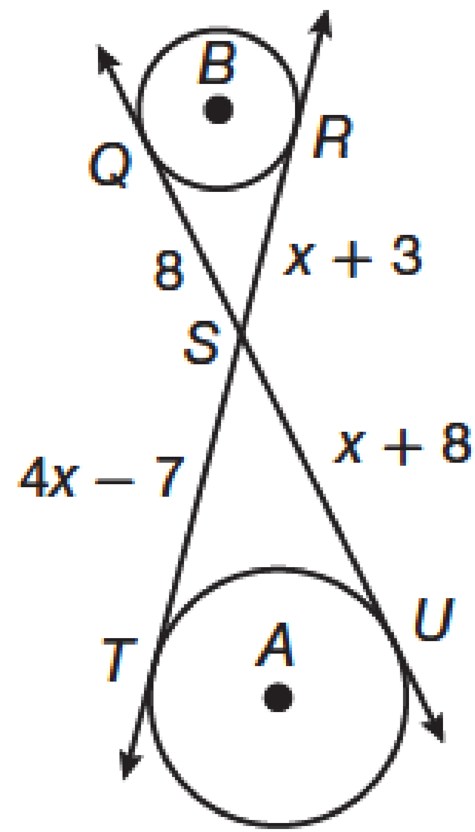
$x = 3$



Substituting the value of *x* back into the expressions in the diagram, *IH* and *KL* both equal 1.5 feet.

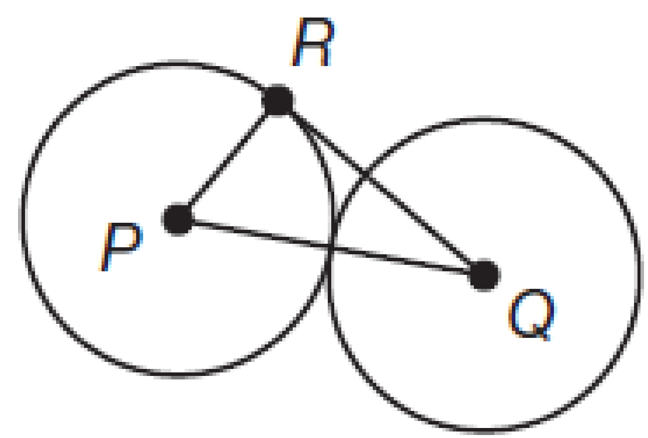
You Try!!!!

a.In the diagram, $\overline{RT}$ and $\overline{QU}$ are tangents to the circles. Find the lengths of $\overline{RS}$, $\overline{ST}$, and $\overline{SU}$.



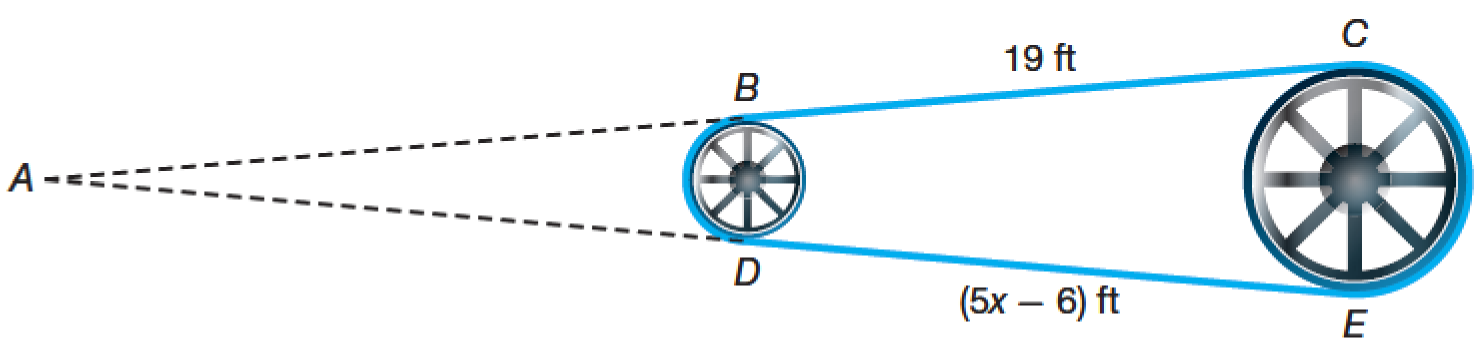
You Try!!!!

b. Determine the area of ΔPQR to the nearest square inch if $\odot P$ and $\odot Q$ are congruent tangent circles with radii of 6 inches each.



You Try!!!!

c. Pulleys: A system of pulleys is set up as shown. Find the value of *x*.



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)