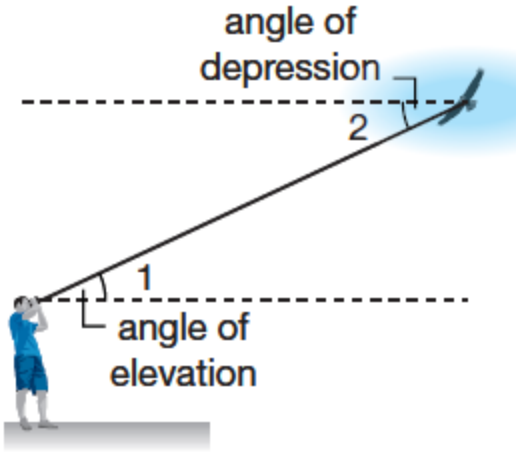
Lesson 73

Applying Trigonometry: Angles of Depression and Elevation

An angle of elevation is the angle formed by a horizontal line and the line of sight to a point above. In the diagram, $\angle 1$ is the angle of elevation. It is the angle to which the bird watcher raises (or elevates) his line of sight from the horizontal to see the bird.

An angle of depression is the angle formed by a horizontal line and a line of sight to a point below. In the diagram, $\angle 2$ is the angle of depression. It is the angle that the bird would lower (or depress) its line of sight from the horizontal to see the bird watcher.

You can use trigonometry to find the angle of elevation and the angle of depression.



Example 1 Angle of Elevation

Use the angle of elevation between the kite and the child to find the horizontal distance between the kite and the child.

SOLUTION

The horizontal distance is x.

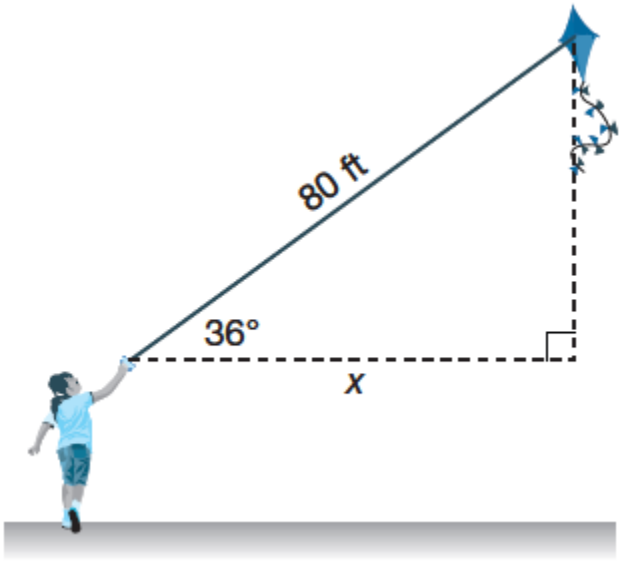
It is adjacent to the angle of elevation.

We also know the hypotenuse, so we should use the cosine ratio.

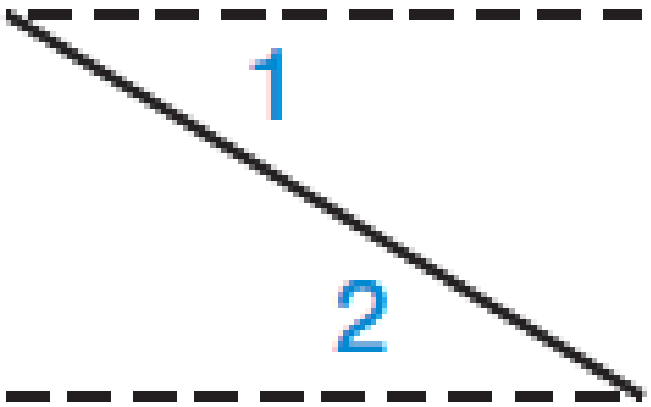
$$\cos 36 = \frac{adjacent}{hypotenuse}$$

$$\cos 36 = \frac{x}{80}$$

$$x \approx 64.72\ ft$$



The horizontal lines used to measure the angles of elevation and depression are parallel, so the angle of elevation, $\angle 2$, and the angle of depression, $\angle 1$, are congruent, because they are alternate interior angles.

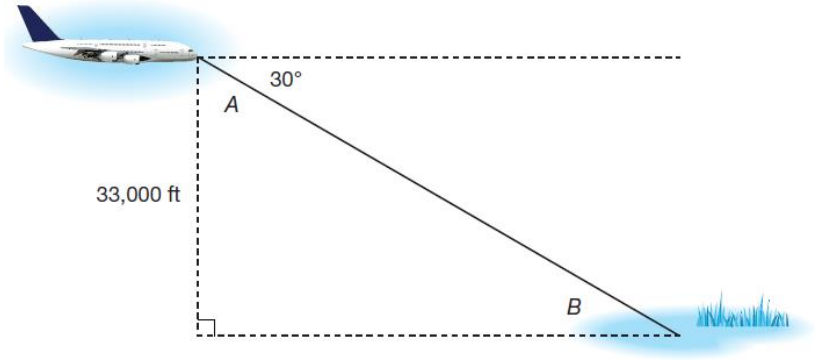


Example 2 Angle of Depression

The pilot in a plane cruising at 33,000 feet sees a lake. If the angle of depression from the plane to the lake is 30°, how far is the plane from the lake?

SOLUTION

Since the angle of elevation is congruent to the angle of depression, the angle of elevation is 30°.



$$\sin 30 = \frac{opposite}{hypotenuse}$$

$$\sin 30 = \frac{33000}{x}$$

$$x = 66{,}000\ ft$$

The plane is 66,000 feet from the lake.

Example 3 Angles of Elevation from Multiple Points

A window washer is working 60 feet above the ground. About how far away is each person pictured at street level?

SOLUTION

In both cases, the unknown value is the hypotenuse of the triangle, and the side opposite the angle of elevation is known, so the sine ratio should be used.

$$\sin A = \frac{opposite}{hypotenuse}$$

Find the length of $\overline{AC}$.

$$\sin 40 = \frac{60}{x}$$

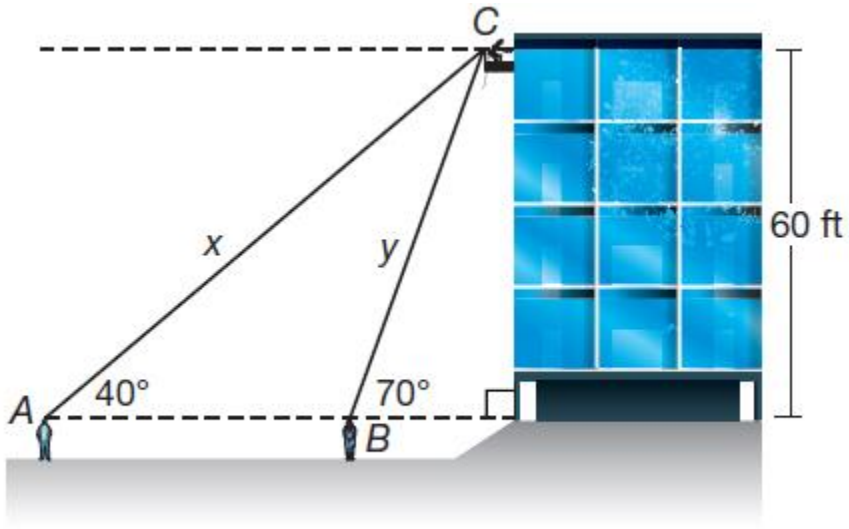
$$x \approx 93.34$$

Find the length of $\overline{BC}$.

$$\sin 60 = \frac{60}{y}$$

$$y \approx 63.85$$

Person *B* is about 64 feet away from the window washer, and person *A* is about 93 feet away.



Example 4 Application: Surveying

A surveyor on the Credit Union building measures an angle of elevation of 15° to the top of the Business Park building. The Credit Union building is 600 feet tall, and the Business Park building is 800 feet tall. What is the distance between the buildings, to the nearest foot?

SOLUTION

Draw a triangle showing the difference in height between the buildings and the angle of elevation.

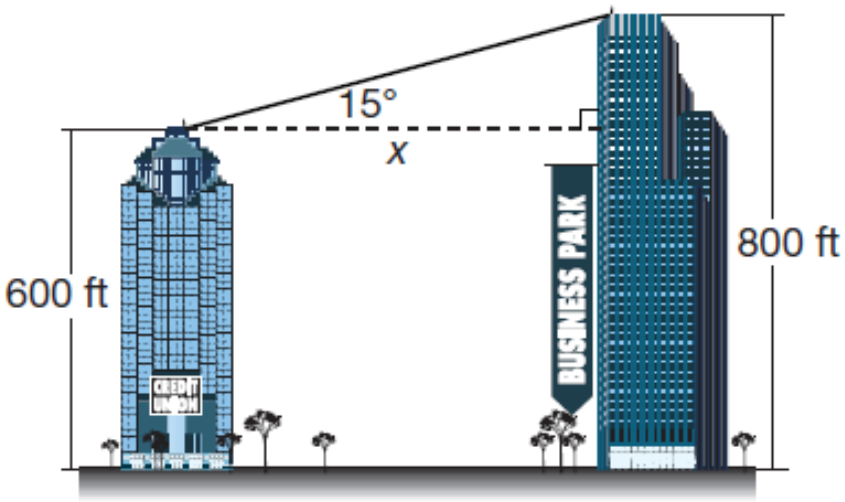
Find the difference in height. 800 – 600 = 200.

Find the horizontal distance between the buildings using the tangent ratio.

$$\tan 15 = \frac{opposite}{adjacent}$$

$$\tan 15 = \frac{200}{x}$$

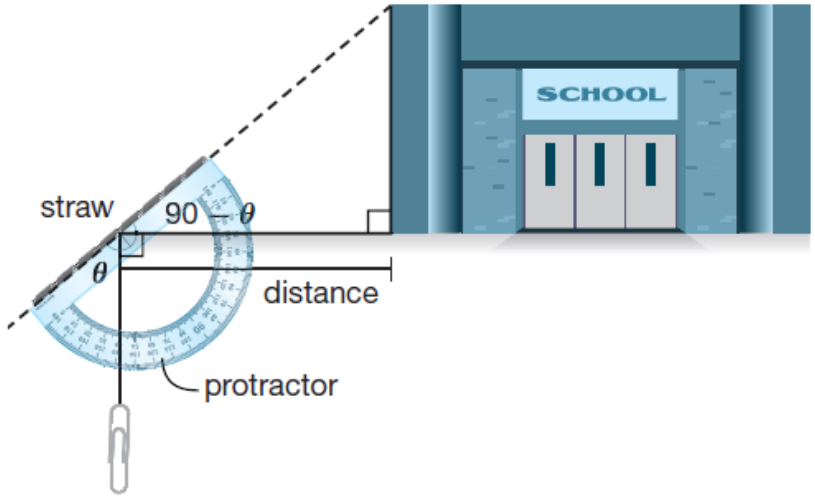
$$x \approx 746\ ft$$



Exploration Making and Using a Hypsometer

In this exploration, you will construct a hypsometer. A hypsometer is used to measure an angle of elevation or an angle of depression.

1. To make a hypsometer, you will need a drinking straw, a protractor, string, a paperclip (or small weight), and tape. First, attach the straw to the straight edge of the protractor with tape. Tie one end of the string to the paper clip. Attach the other end of the string to the center point of the protractor.



Exploration Making and Using a Hypsometer

2. Work with a partner to use the hypsometer to measure the height of your school. While one person looks through the straw at the top of the school, the other person measures the angle of the string on the protractor. As you can see in the diagram, the angle that the string makes with the protractor will be the complement of the angle of elevation.

3. Measure the distance to the school.

Caution

When measuring the distance to the school, be sure to measure the distance from the point where you used the hypsometer to the point on the school directly below where you viewed the school's height, and not the point on the school closest to where you used the hypsometer.

Exploration Making and Using a Hypsometer

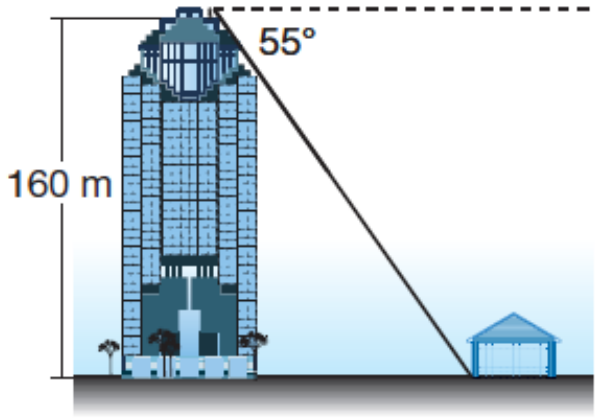
4. Use the tangent function to find the height of the school. The adjacent side is the distance between you and the school. The opposite side is the school's height. Remember to add the height of your eyes from the ground to the calculated height of the school to get a more accurate measurement. After you calculate the height of the school, compare your answer with those of other students.

5. Use the same method to find the heights of trees or other buildings.

You Try!!!!!

a. Standing 300 feet away from a building, the angle of elevation to the roof is 67°. Find the height of the building to the nearest foot.

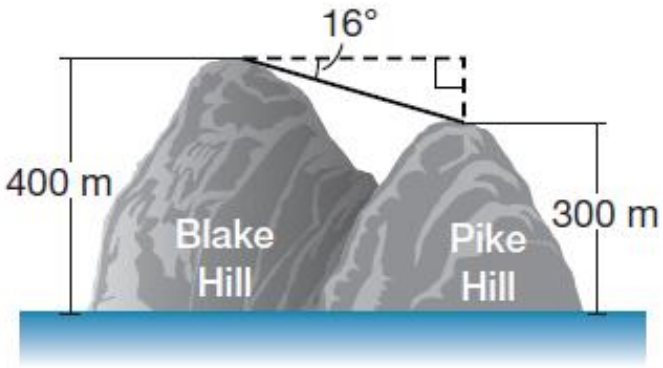
b. A family sees their house from a window of a tall building. The angle of depression is 55°. The window where the family is standing is 160 meters above the street. How far away is their house from the window, to the nearest meter?



You Try!!!!!

c. Jocelyn sees a hot-air balloon at an angle of elevation of 30°. Anthony sees the same hot-air balloon at an angle of elevation of 50°. The balloon is 140 meters off the ground. They are starting on the same side of the balloon. How far apart are Jocelyn and Anthony, to the nearest meter? Who is farther away from the balloon?

d. Surveying: A surveyor on the top of Blake Hill measures an angle of depression of 16° to the top of Pike Hill. The height of Blake Hill is 400 meters, and the height of Pike Hill is 300 meters. What is the distance between the hilltops, to the nearest meter?



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1-30 (Do the starred ones first)