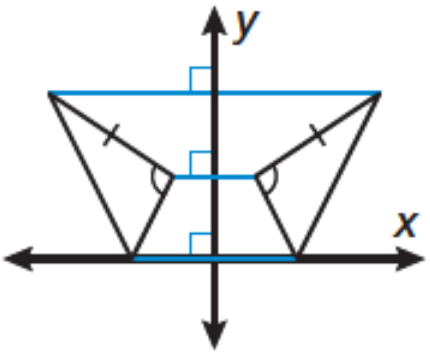
Lesson 74

Reflections

A reflection is a transformation that reflects every point in a figure over a given line. After reflection, the image of the figure is congruent to the preimage, but has a different orientation.

Property of Reflection – A reflection is an isometry, meaning the preimage and its reflected image have the same shape and size.

To reflect a point across a horizontal or vertical line, imagine that the line is a mirror, and visualize the reflected location of the point. The figure shows a triangle reflected over the *y*–axis.



Example 1 Reflecting Across an Axis

Reflect ΔABC across the y–axis. Find the coordinates of the vertices of the reflected image and write the transformation in mapping notation.

SOLUTION

Imagine each point reflected across a mirror sitting on the y–axis. Each point will end up opposite from where it is now in Quadrant II.

The y–coordinates will not change, but the signs of the x–coordinates are reversed. Each point (x, y) will be mapped to $(-x, y)$.

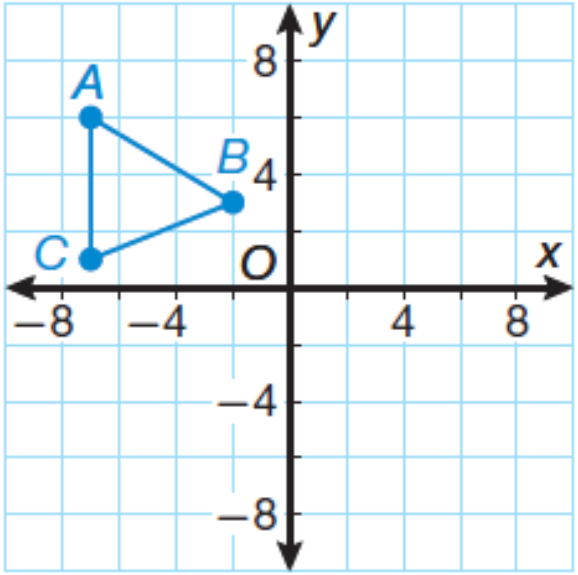
In mapping notation:

T: $(x, y) \rightarrow (-x, y)$.

T: $A(-7, 6) \rightarrow A'(7, 6)$

T: $B(-2, 3) \rightarrow B'(2, 3)$

T: $C(-7, 1) \rightarrow C'(7, 1)$



Example 2 Reflecting Across a Horizontal Line

Reflect the rectangle *STUV* across the line $y = 4$. Identify the coordinates of the vertices of the reflected image.

SOLUTION

After a reflection, each point will be the same distance from the mirror as it is now.

For example, *S* is 4 units away from the mirror.

After reflection, it will still be 4 units away, but in the opposite direction.

So it is reflected to (–2, 0), where it is 4 units from $y = 4$.

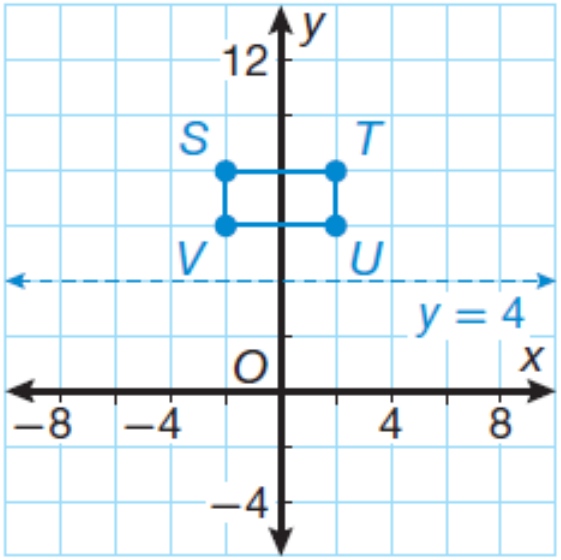
In mapping notation:

T: $(x, y) \rightarrow (x, -y + 8)$.

T: $T(2, 8) \rightarrow T'(2, 0)$

T: $U(2, 6) \rightarrow U'(2, 2)$

T: $V(-2, 6) \rightarrow V'(-2, 2)$



Notice that when a point is reflected across a horizontal line, its x-coordinate does not change. When a point is reflected across a vertical line, its y-coordinate does not change.

To find the reflection of a point across any line in the coordinate plane, draw a perpendicular line from the point to the line of reflection. The point's reflection will be equidistant from the line of reflection on both sides.

Example 3 Reflecting Across a Line

Reflect quadrilateral *JKLM* across the line $y = x$. Identify the coordinates of the vertices of the reflected image.

SOLUTION

The perpendicular line to $y = x$ is $y = -x$.

In the second diagram, the perpendicular line from *M* to its reflection, *M'* is shown.

When a point is reflected over the line $y = x$, it follows the transformation:

$T: (x, y) \rightarrow (y, x)$.

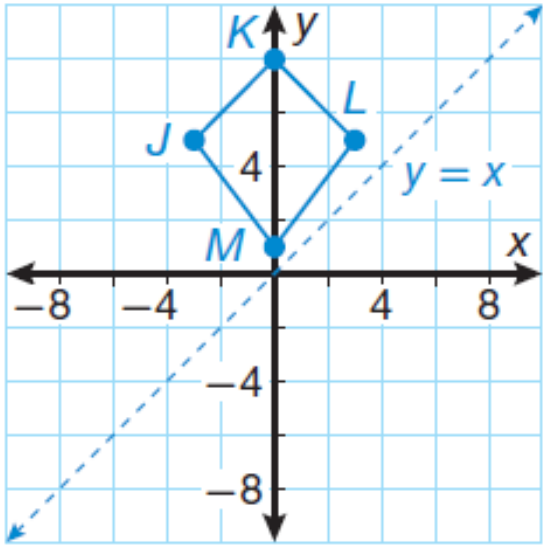
Apply this to the vertices of the quadrilateral shown.

$T: J(-3, 5) \rightarrow J'(5, -3)$

$T: K(0, 8) \rightarrow K'(8, 0)$

$T: L(3, 5) \rightarrow L'(5, 3)$

$T: M(0, 1) \rightarrow M'(1, 0)$



Example 4 Application: Visual Arts

Marina is creating a work of art using part of a photograph and its reflection. In a coordinate grid, the corners of the photograph fragment are located at (–3, 2), (2, 8), and (10, 2). Reflect the fragment across the line $y = 2$.

SOLUTION

Points that lie on $y = 2$ do not move at all, since they are on the line of reflection.

The third point (2, 8) is 6 units from the line of reflection.

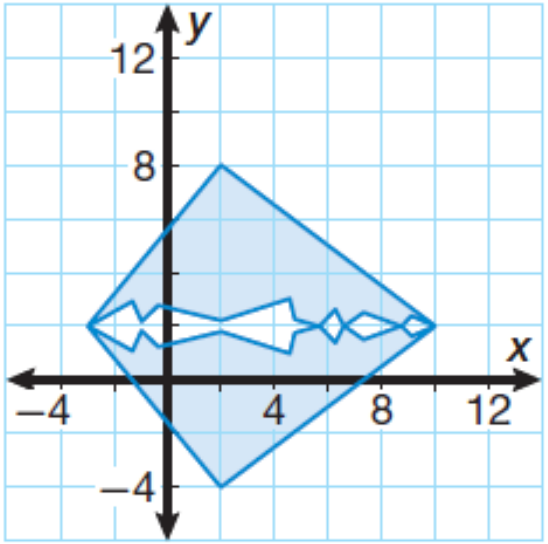
When it is reflected, it will lie 6 units from the line of reflection on its other side, at (2, –4).

The transformation is: T: $(x, y) \rightarrow (x, 4 - y)$.

Verify that the other 2 points do not move.

T: (–3, 2) → (–3, 4 – 2) = (–3, 2)

T: (10, 2) → (10, 4 – 2) = (10, 2)



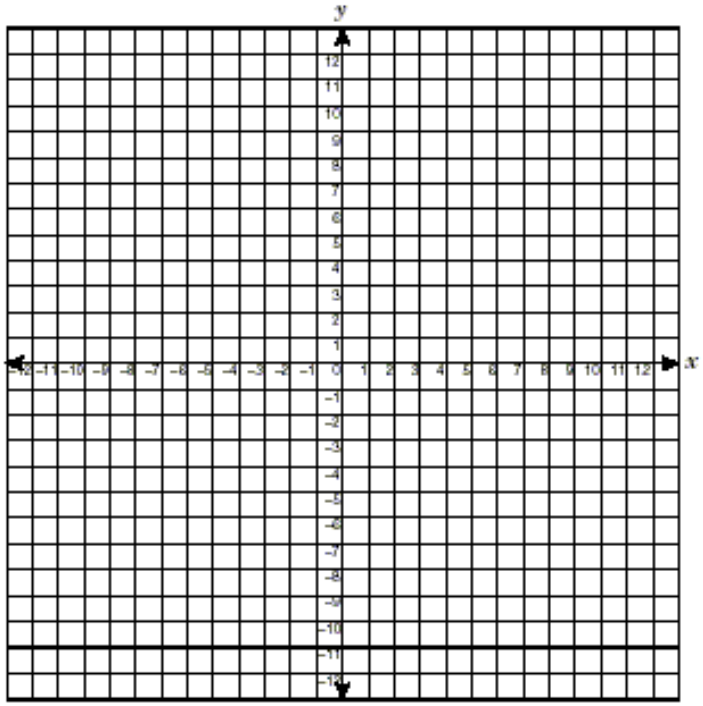
You Try!!!!

Rectangle *ABCD* has vertices at *A*(1, 1), *B*(5.5, 1), *C*(5.5, 3.5), and *D*(1, 3.5). Reflect *ABCD* as described in parts a through c.

a.Reflect *ABCD* across the *y*–axis.

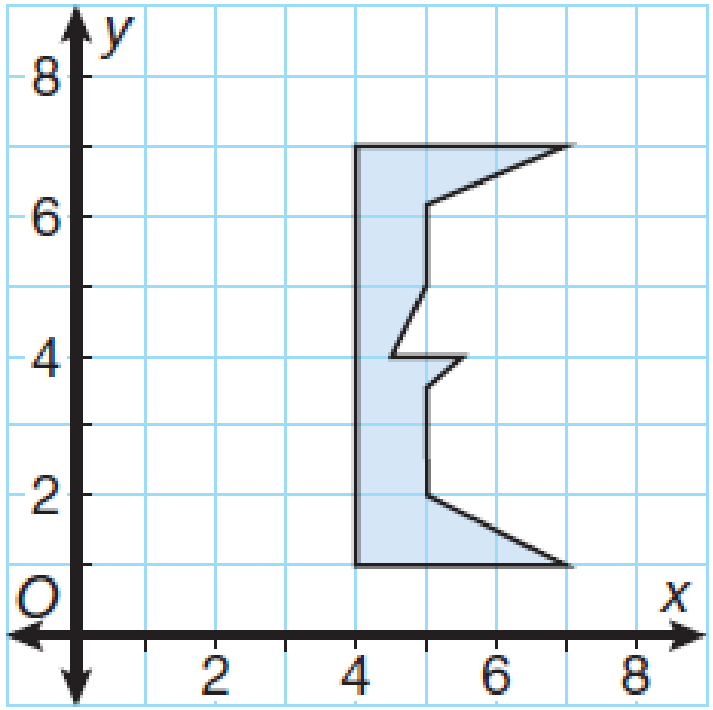
b.Reflect *ABCD* across the line $y = 2$.

c.Reflect *ABCD* across the line $y = x$.



You Try!!!!

d.Visual Arts: This figure shows half of an optical illusion. Complete the figure by reflecting it across the line $x = 4$.



Assignment

Page 492

Lesson Practice (Ask Mr. Heintz)

Page 492

Practice 1-30 (Do the starred ones first)