Lesson 75

Writing the Equation of a Circle

To analyze a circle on a coordinate plane, it is necessary to derive an equation for the graph of a circle. A simple version of this equation can be found by looking at circles that are centered at the origin.

Math Language

The **unit circle** is a circle with a radius of 1, centered at the origin. The unit circle is frequently used in trigonometry.

Example 1 Analyzing a Circle Centered at the Origin

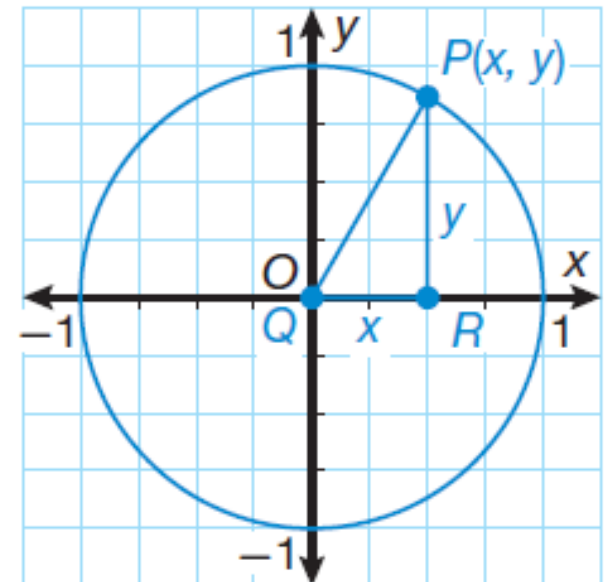Suppose $P(x, y)$ is a point on the circle with a 1-unit radius that is centered at the origin.

What is QP? How is QP related to x and y?

SOLUTION

Since $\overline{QP}$ is a radius of the circle, $QP = 1$.

In the diagram, it is shown that $\triangle PQR$ is a right triangle. Therefore, by the Pythagorean Theorem, $x^2 + y^2 = 1$

The equation given above, $x^2 + y^2 = 1$, gives a relationship between x and y that is true of all points that lie on the circle. Therefore, it is an equation for the circle with a radius of 1 that is centered at the origin. Any circle centered at the origin has an equation given by $x^2 + y^2 = r$, where r is the radius.

If the circle is not centered at the origin, this alters the equation. Examine the circle centered at (2, 0) with a radius of 2. For the point (x, y), the distance from the center of the circle to (x, y) is $\sqrt{(x-2)^2 + (y-0)^2} = 2$, by the distance formula. So the equation for this circle is $(x-2)^2 + y^2 = 4$.
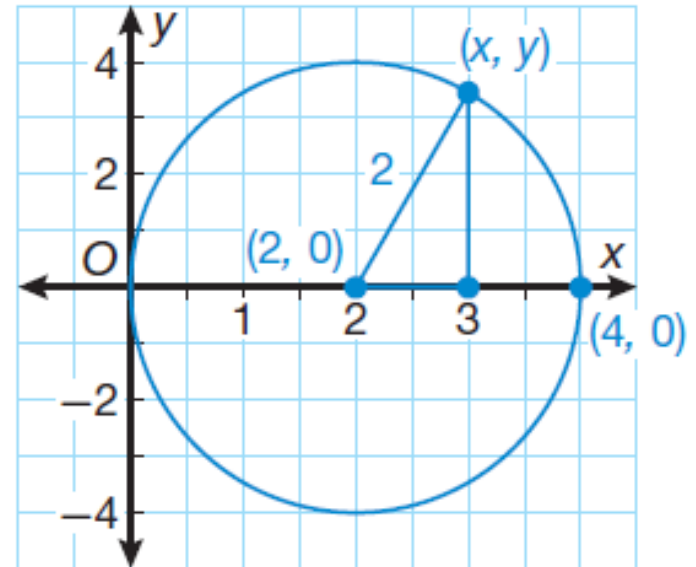
The Equation of a Circle – The equation of a circle is given by the following formula, where (h, k) is the center of the circle and r is the radius of the circle.

$$(x - h)^2 + (y - k)^2 = r^2$$

Example 2 Writing the Equation for a Circle from a Graph

a. If A is a point on $\odot D$, write the equation of $\odot D$.

SOLUTION

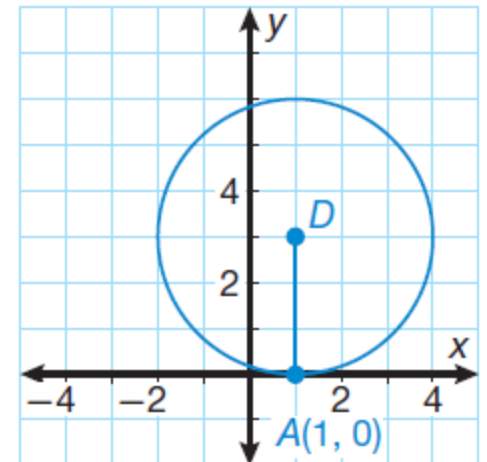The center of the circle is $D(1, 3)$.

Since A is a point on the circle, the radius of the circle is 3 units.

Substitute these values into the equation for a circle.

$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle

$(x - 1)^2 + (y - 3)^2 = 3^2$ Substitute.

Example 2 Writing the Equation for a Circle from a Graph

b. Circle *E* is concentric with $\odot D$ and has a radius of 5. Write the equation of $\odot E$.

SOLUTION

Since $\odot E$ has the same center as $\odot D$, only the radius needs to be changed.

Substitute in the radius of 5 where 3 was in the equation from part a.

$$(x - 1)^2 + (y - 3)^2 = 5^2$$

Example 3 Graphing a Circle Given its Equation

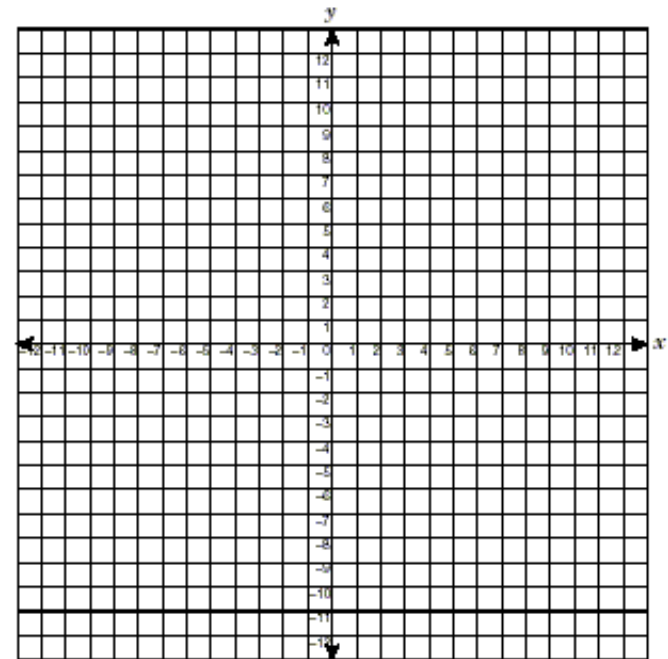a. The equation of $\odot P$ is $x^2 + y^2 = 25$. Graph $\odot P$.

SOLUTION

Since h and k are both 0 in this equation, the circle is centered at the origin.

The radius is $\sqrt{25} = 5$.

To graph $\odot P$, find some simple points.

The endpoints of the horizontal and vertical radii are (0, 5), (5, 0), (–5, 0), and (0, –5).

Using these points as guides, draw the circle, as shown in the diagram.

Example 3 Graphing a Circle Given its Equation

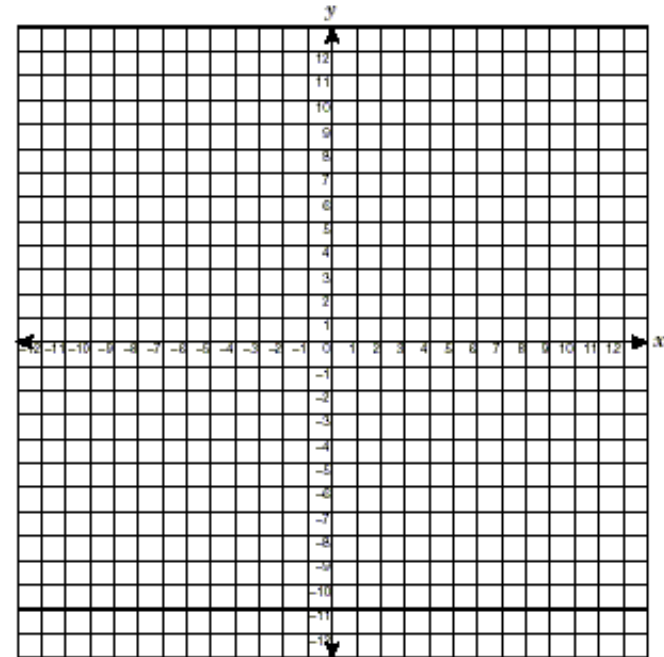b. The equation of $\odot Q$ is $(x - 2)^2 + (y + 1)^2 = 16$. Graph $\odot Q$.

SOLUTION

Rewrite the equation in the form $(x - h)^2 + (y - k)^2 = r^2$.

$$(x - 2)^2 + \big(y - (-1)\big)^2 = 4^2$$

The center of $\odot Q$ is (2, −1) and its radius is 4.

Four points on $\odot Q$ are the endpoints of the horizontal and vertical radii: (2, 3), (6, −1), (2, −5), and (−2, −1).

Example 4 Application: Astronomy

This coordinate grid shows a satellite's orbit around Earth, which is located at the origin. If the satellite's distance from Earth is 23,000 miles, write an equation that describes the satellite's circular orbit.

SOLUTION

The center of the orbit is the origin, and its radius is OP = 23,000. Therefore:

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 0)^2 + (y - 0)^2 = 23000^2$$
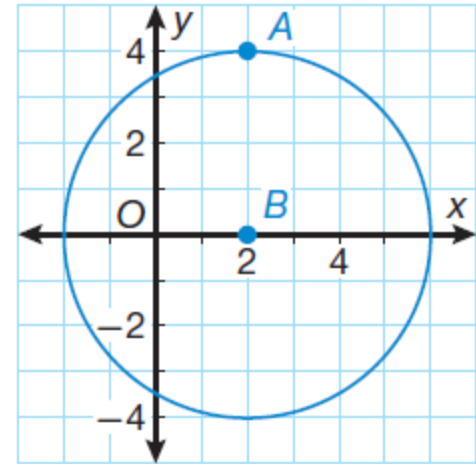$$x^2 + y^2 = 529000000$$
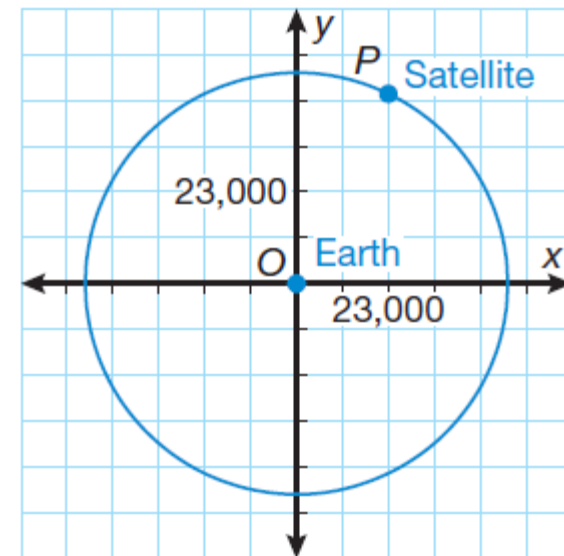
You Try!!!!

a. If $M(x, y)$ is a point on a circle centered at the origin with a radius of 3, what is PM, and what is the equation of the circle?

b. Write an equation to relate all the x– and y– coordinates of points that lie on $\odot A$ with a radius of $\sqrt{2}$, which is centered at the origin.
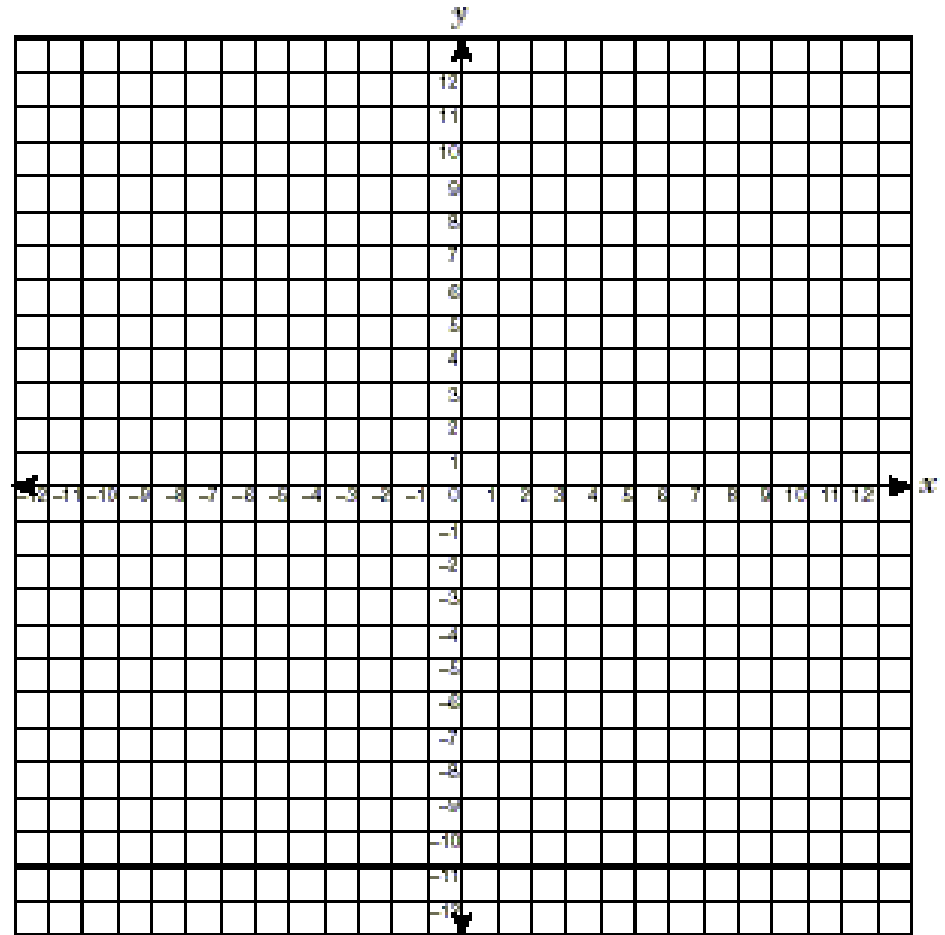
You Try!!!!

c. Write the equation for ⊙B.

d. Circle C is concentric with ⊙B and has a radius of 3.5. Write the equation of ⊙C.
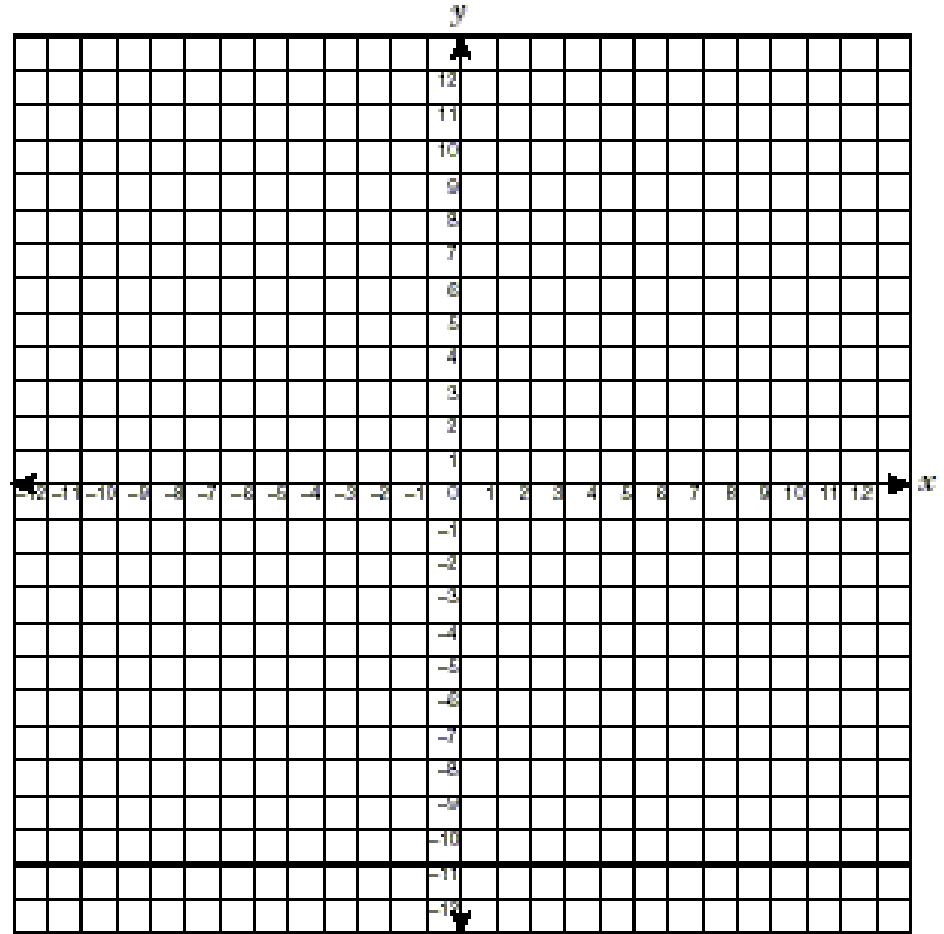
You Try!!!!

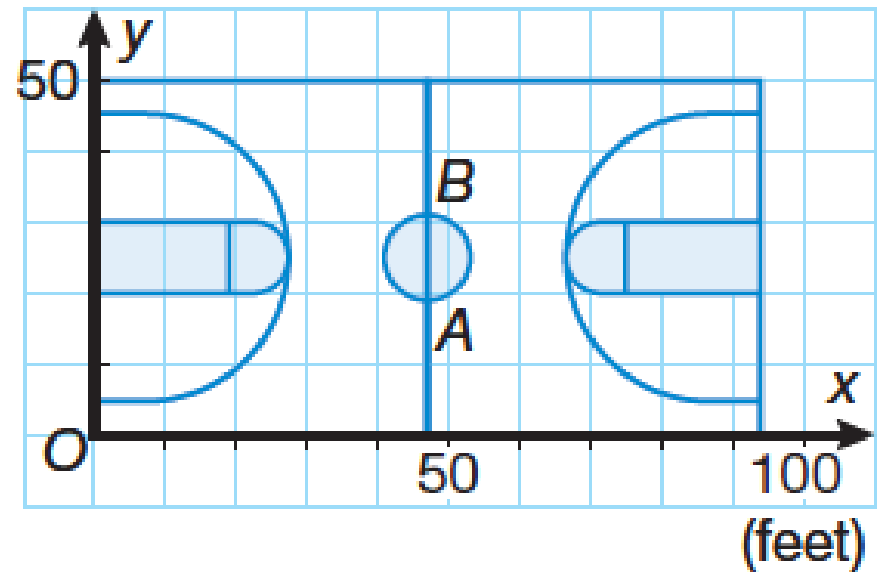e. The equation of $\odot D$ is $x^2 + y^2 = 6.25$. Graph $\odot D$.

You Try!!!!

f. The equation of $\odot E$ is $(x + 1)^2 + (y - 3)^2 = 4$. Graph $\odot E$.

You Try!!!!

g. Sports: This coordinate grid shows the position of the lines on a basketball court. The center circle crosses the half court line at A(47, 19) and B(47, 31), so $\overline{AB}$ is a diameter. What is the equation of the center circle?

Assignment

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    Practice 1-30 (Do the starred ones first)