Lesson 7

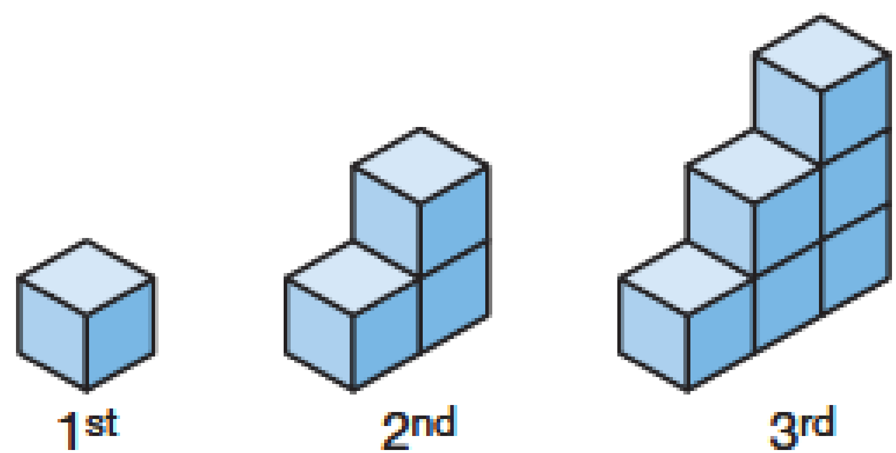
Using Inductive Reasoning

Inductive reasoning – the process of reasoning that a rule or statement is true because several specific cases are true. Inductive reasoning can be used to formulate a conjecture about something.

Conjecture – a statement that is believed to be true. If a conjecture can be proven, it becomes a theorem.

Example 1. Formulating a Conjecture.

Look at the progression of the pattern and formulate a conjecture regarding the number of blocks there will be in the fifth arrangement of this series.

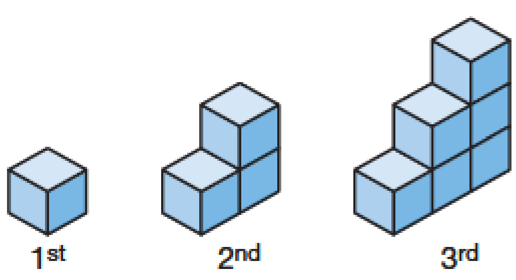


Example 1. Formulating a Conjecture.

SOLUTION

Each successive step of the series adds a row of blocks to the bottom of the figure that is one block longer than the previous row of blocks. For example, the second arrangement has a bottom row that is 2 blocks long, so to make the third one, a row of 3 blocks is added to the bottom.

To continue, the fourth step would have a row of 4 blocks added for a total of 10, and the fifth would have 5 more blocks added for a total of 15. The statement, “There will be 15 blocks in the fifth step of this pattern,” is a conjecture.



Instead of formulating a conjecture by looking at data, it may be necessary to test a conjecture using given data. If even one example can be found that does not support the conjecture, then the conjecture must be incorrect.

Counterexample – An example that does not support the conjecture.

Example 2. Testing a Conjecture.

a. Michelle made the conjecture, "The expressions $6n + 1$ and $6n - 1$ will always result in two prime numbers." Show that this conjecture is true for $n = 1$, 2, and 3, but not true for $n = 4$.

SOLUTION

For $n = 1$: $6(1) + 1 = 7$ and $6(1) - 1 = 5$; both are prime.

For $n = 2$: $6(2) + 1 = 13$ and $6(2) - 1 = 11$; both are prime.

For $n = 3$: $6(3) + 1 = 19$ and $6(3) - 1 = 17$; both are prime.

But for $n = 4$: $6(4) + 1 = 25$ and $6(4) - 1 = 23$; 25 is not prime.

Since we have found one case in which the conjecture is incorrect, we can conclude that the conjecture is false.

Example 2. Testing a Conjecture.

b. Maria looks at the diagram below and conjectures that the number of triangles in the figure is given by the expression $2n + 1$. Is this conjecture true for the four steps of the pattern shown below?

SOLUTION

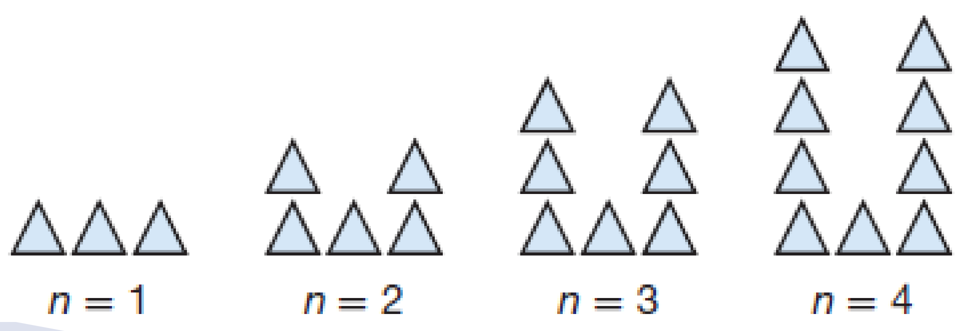
Yes, the conjecture is true for all 4 steps.

For $n = 1$, $2(1) + 1 = 3$

For $n = 2$, $2(2) + 1 = 5$

For $n = 3$, $2(3) + 1 = 7$

For $n = 4$, $2(4) + 1 = 9$



Even though conjectures are not proven, scientists often use conjectures to describe real-world phenomena. These conjectures are carefully studied and tested, but it is often difficult to prove them formally.

Example 3 Application: Research

A researcher studying crows for several years made the observation that every crow she studied was black. Her research assistant made this conjecture: “All crows are black.” How can this conjecture be tested? Can it be proved?

SOLUTION

The conjecture can be tested by observing as many crows as possible. If even one crow is found that is not black, then the conjecture is disproved. The only way to prove this conjecture is to observe every crow. If every crow can be studied, and they are all black, then the conjecture is true. However, it is impossible to study every crow that exists, so the conjecture cannot be proved.

You Try!!!!

a. Formulate a conjecture about how the next step in this pattern would be found: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …

The value that is found next in the sequence is found by adding the previous two values.

b. Test the conjecture that every even integer 4 through 14, can be written as the sum of two prime numbers.

True: $4 = 2 + 2, 6 = 3 + 3, 8 = 3 + 5, 10 = 3 + 7\ or\ 5 + 5, 12 = 5 + 7, 14 = 7 + 7\ or\ 3 + 11$

c. How might you disprove the conjecture below?

Apples, pears, lemons, and peaches all grow on trees, therefore all fruits grow on trees.

Find an example of a fruit that does not grow on trees.

Assignment

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Practice 1–30 (Do the starred ones first)