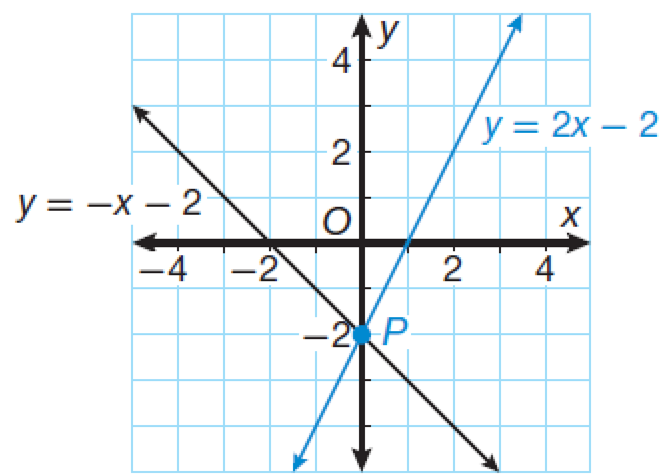
Lesson 81

Graphing and Solving Linear Systems

A system of equations is a set of two or more equations that have two or more variables. A system of linear equations can be solved algebraically or by graphing the lines.

If the lines are graphed, the solution to the system is the coordinates of the point where the lines intersect. In this example, the solution to the system is point *P*. To solve a system algebraically, solve both equations for the same variable and use substitution.



Example 1 Solving Linear Systems Algebraically

Solve this system of equations algebraically.

$$y = \frac{1}{2}x - 1 \qquad\qquad y = -\frac{3}{2}x + 3$$

$$\frac{1}{2}x - 1 = -\frac{3}{2}x + 3$$

$$+\frac{3}{2} \qquad\qquad +\frac{3}{2}x$$

$$2x - 1 = 3$$

$$+1 \;\; +1$$

$$2x = 4$$

$$\frac{2x}{2} = \frac{4}{2}$$

$$x = 2$$

$$y = \frac{1}{2}(2) - 1$$

$$y = 1 - 1$$

$$y = 0$$

So the solution to this system is the ordered pair (2, 0).

Example 2 Solving Linear Systems Graphically

Estimate the solution to this linear system by graphing the lines.

$y = -\frac{1}{2}x - 3$ $\qquad$ $y = \frac{3}{2}x + 1$

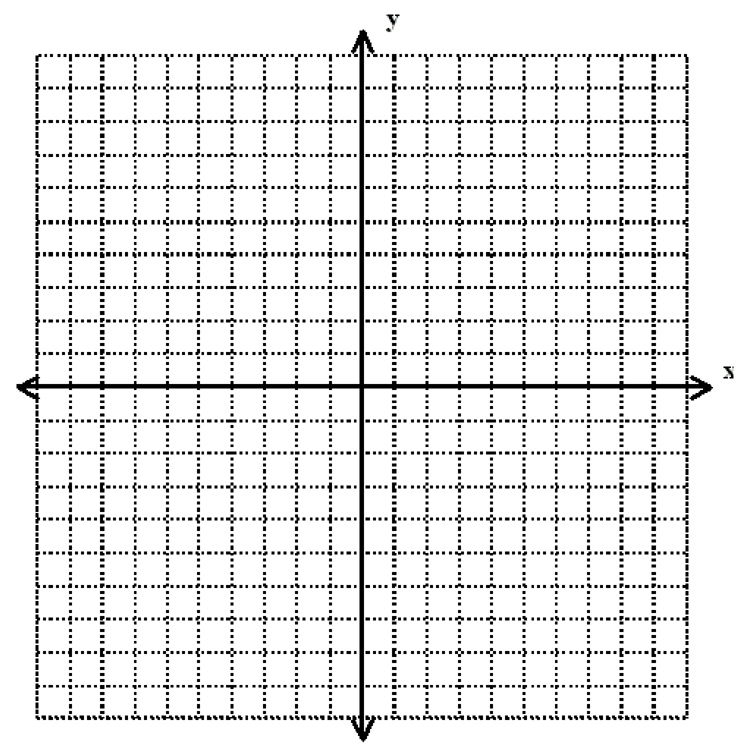
SOLUTION

Graph each line on a coordinate grid.

The lines appear to intersect at the point (−2, −2).

Check the answer by substituting

x=−2 and y=−2 into both of the original linear equations.



If two lines are parallel, they do not intersect. A system of equations that represents two or more parallel lines has no solution.

Example 3 Analyzing Unsolvable Systems

Graph this linear system to determine if there is a solution.

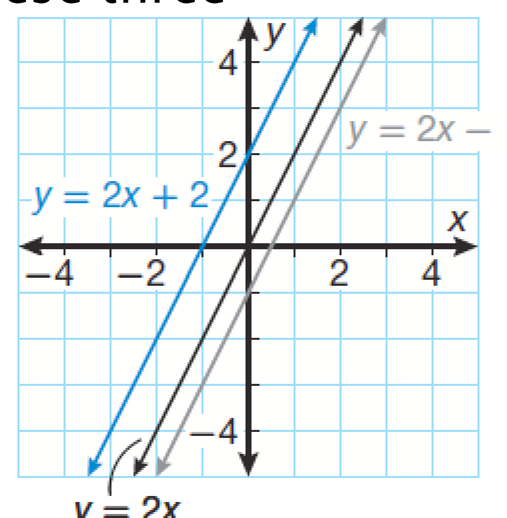
$y = 2x - 1$ $\quad y = \frac{4}{2}x + 2$ $\quad 2y - 2 = 4x - 2$

SOLUTION

Simplify each equation, then graph the lines on the same coordinate grid.

$y = 2x - 1$ $\quad y = 2x + 2$ $\quad y = 2x$

Since all three lines have the same slope, they are parallel. Graphing them demonstrates that they have no point of intersection. Since all three lines are parallel, there is no solution to the system of linear equations that consists of these three equations.



Note that solving a system of equations graphically is not as accurate as solving the same system algebraically.

Math Reasoning

Estimate In Example 3, suppose the third line had slope 2.1. Would this line intersect each of the other two lines? If yes, in which quadrant would the lines intersect?

Example 4 Application: Economics

An economist is trying to determine the optimum price for a new product. He knows the supply of the product is represented by the function $y = \frac{2}{3}x + 50$ and the demand curve for the product is represented by the function $y = -\frac{1}{3}x + 200$, where *y* is the price of the product and *x* is the number of units sold. What is the optimum price of the product? How many units will sell at this price?

SOLUTION

Solve the system algebraically by substituting for *y*.

$$\frac{2}{3}x + 50 = -\frac{1}{3}x + 200$$

$$+\frac{1}{3}x \qquad\qquad +\frac{1}{3}x$$

$$x + 50 = 200$$

$$-50 \quad -50$$

$$x = 150$$

$$y = \frac{2}{3}(150) + 50$$

$$y = 100 + 50$$

$$y = 150$$

The solution is (150, 150). This means that the optimum price of the product is \$150 and 150 units will be sold at this price.

Example 4 Application: Economics

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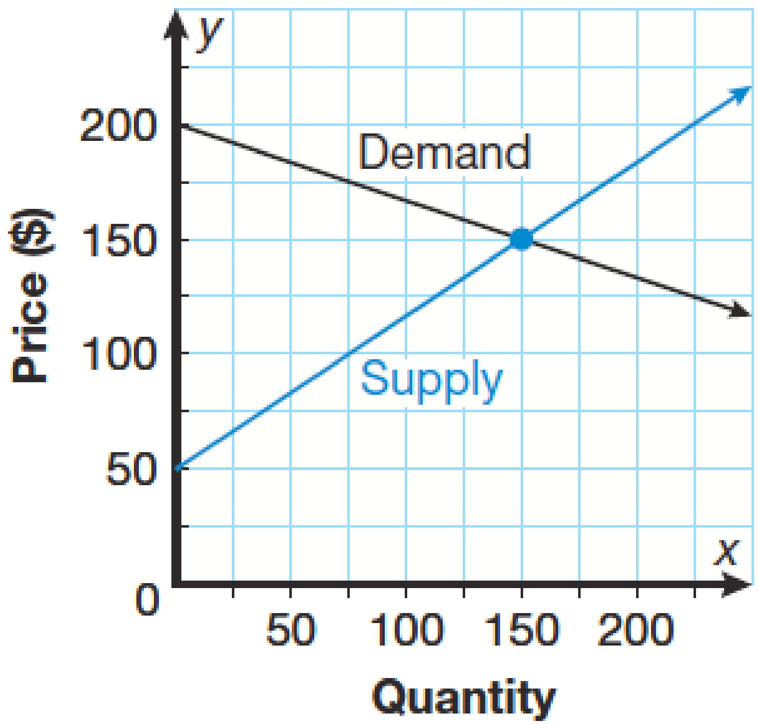


You Try!!!!

a.Solve the system of equations algebraically.

$y = \frac{2}{3}x - 8$ $\qquad$ $y = \frac{1}{4}x + 2$

$(24, 8)$

b.Solve the system of equations algebraically.

$y = -\frac{2}{3}x - 3$ $\qquad$ $y = \frac{1}{2}x + 2$

$\left(-\frac{30}{7}, -\frac{1}{7}\right)$

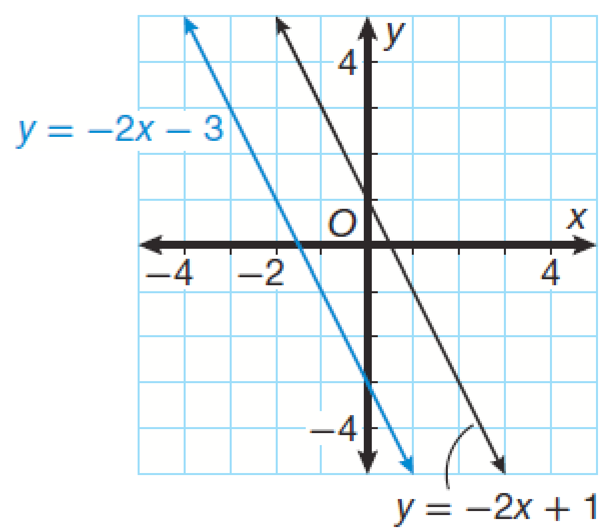
You Try!!!!

c.Solve the system of equations by graphing.

$y = 3x + 4$ $\quad$ $y = -x + 8$

$(1, 7)$

d. Determine if there is a solution for this system. If not, explain why.



No. They are parallel.

You Try!!!!

e.The supply curve for a product is represented by $y = 2x + 20$ and the demand curve for the product is represented by $y = -\frac{1}{2}x + 80$ where y is the price of the product, and x is the number of units sold. What is the optimum price of the product and how many units will be sold at this price?

$68; 24 Units

Assignment

Page 534

Lesson Practice (Ask Mr. Heintz)

Page 535

Practice 1–30 (Do the starred ones first)