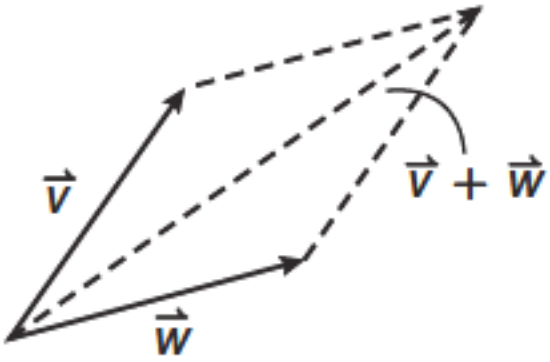
Lesson 83

Vector Addition

You have added vectors that are in the same or opposite directions, but how are vectors added when their directions are not so simple? There are two commonly used methods for adding any two vectors together. The parallelogram method is a method of adding two vectors by drawing a parallelogram using the vectors as two of the consecutive sides; the sum is a vector along the diagonal of the parallelogram.

In the diagram, both $\vec{v}$ and $\vec{w}$ are drawn with the same initial point. A vector parallel to $\vec{v}$ and a vector parallel to $\vec{w}$ complete the parallelogram, and the diagonal is the resultant vector.



Example 1 Using the Parallelogram Method to Add Vectors

Use the parallelogram method to add the two vectors.

$\vec{a} = \langle 2, 3 \rangle \quad \vec{b} = \langle 4, -1 \rangle$

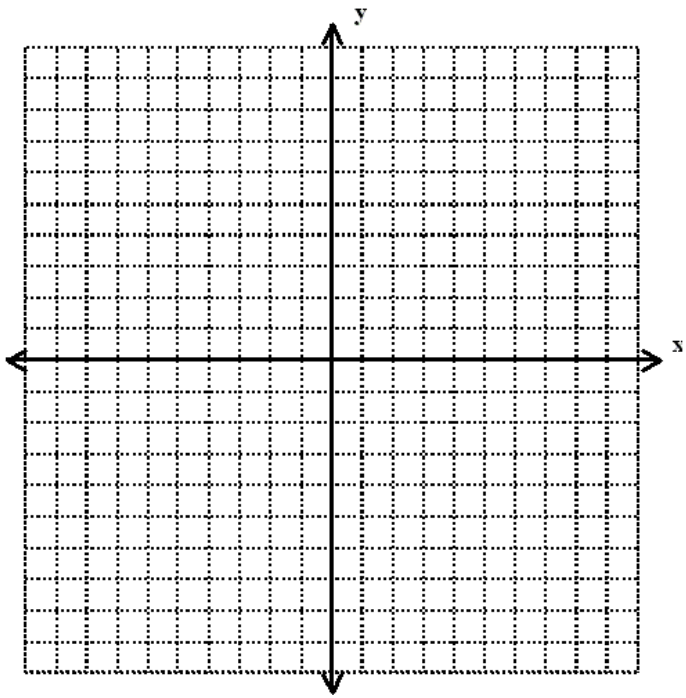
SOLUTION

Draw the vectors on a coordinate plane starting at the origin. Complete the parallelogram by drawing identical parallel vectors that start at the terminal points of the original vectors.

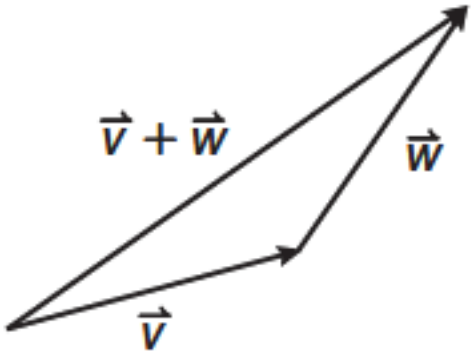
Draw the diagonal of the parallelogram from the origin. This diagonal is the resultant vector.

Counting its vertical and horizontal change shows that the component form for the resultant vector is ⟨6, 2⟩.

$$\vec{a} + \vec{b} = \langle 2,3 \rangle + \langle 4, -1 \rangle$$
$$\vec{a} + \vec{b} = \langle 2 + 4, 3 + (-1) \rangle$$
$$\vec{a} + \vec{b} = \langle 6, 2 \rangle$$



The head-to-tail method is a method of adding two vectors by placing the tail of the second vector on the head of the first vector. The sum is the vector drawn from the tail of the first vector to the head of the second vector. One benefit to using this method is that while the parallelogram method can only be used to add two vectors, the head-to-tail method can be used to add any number of vectors.



Example 2 Adding Vectors Head-to-Tail

Add the vectors $\vec{c} = \langle 5,4 \rangle, \vec{d} = \langle -2,3 \rangle, and\ \vec{e} = \langle -1,-1 \rangle$ using the head-to-tail method.

SOLUTION

First, draw a diagram. The first vector should begin at the origin. The vectors can be placed on the diagram in any order, but each vector must start at the terminal point of the vector before it.

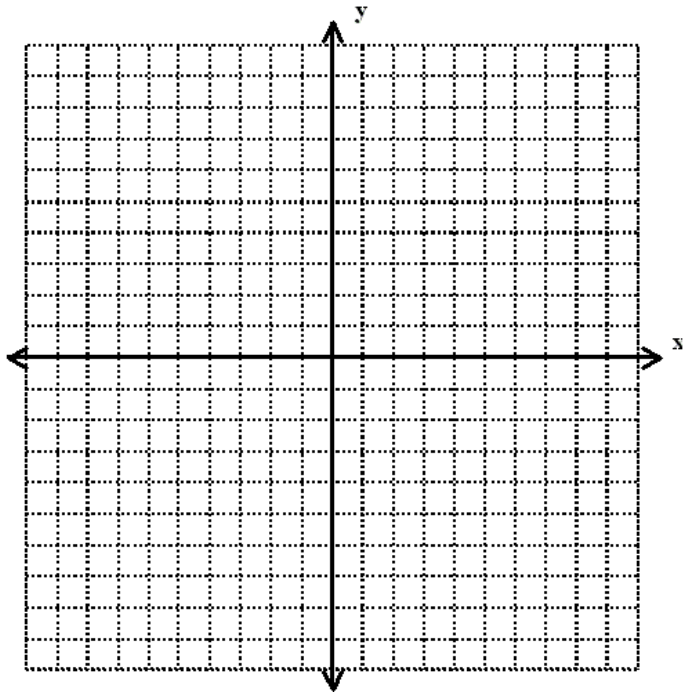
The diagram shows that the resultant vector is

$$\vec{c} + \vec{d} + \vec{e} = \langle 2,6 \rangle$$

Adding the vectors by components gives the same answer.

$$\vec{c} + \vec{d} + \vec{e} = \langle 5,4 \rangle + \langle -2,3 \rangle + \langle -1,-1 \rangle$$

$$\vec{c} + \vec{d} + \vec{e} = \langle 2,6 \rangle$$



Remember that the direction of a vector must also be specified. To find a vector's direction, measure the angle it forms with the horizontal. By convention, when a vector's direction is given with an angle, the angle is measured counterclockwise from the positive x-axis.

Example 3 Adding Vectors with Trigonometry

Add the vectors $\vec{a} = \langle 6,0 \rangle \; and \; \vec{b} = \langle 0,4 \rangle$, and find the magnitude and direction of the resultant vector.

SOLUTION

First, add the vectors using the head-to-tail method.

The resultant vector is ⟨6, 4⟩.

Notice that $\vec{a} \; and \; \vec{b}$ make a right angle.

Use the Pythagorean Theorem to find the magnitude of the resultant vector.

$$|\vec{a}|^2 + \left|\vec{b}\right|^2 = \left|\vec{a} + \vec{b}\right|^2$$

$$6^2 + 4^2 = \left|\vec{a} + \vec{b}\right|^2$$

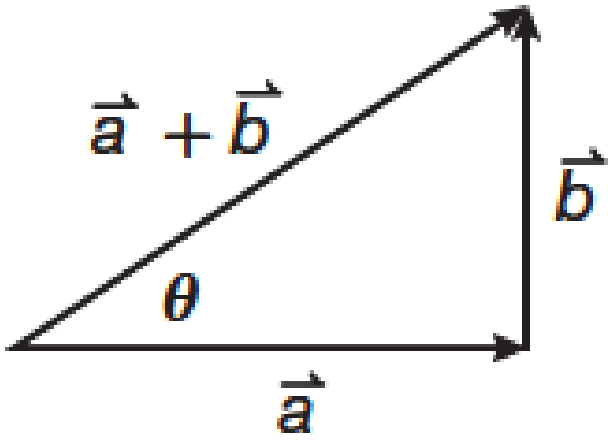
$$\left|\vec{a} + \vec{b}\right| \approx 7.21 \; units$$

Now find the angle θ that the resultant vector makes with the horizontal.

$$\theta = tan^{-1} \frac{opposite}{adjacent}$$

$$\theta = tan^{-1} \frac{4}{6}$$

$$\theta \approx 33.69^{\circ}$$



Example 4 Application: Aviation

A plane has traveled a horizontal distance that can be represented by the vector $\langle 8000, 0\rangle$, and a vertical distance represented by the vector $\langle 0, 4000\rangle$, where the magnitude of both vectors is measured in meters. What is the magnitude of the distance it has traveled to the nearest meter?

SOLUTION

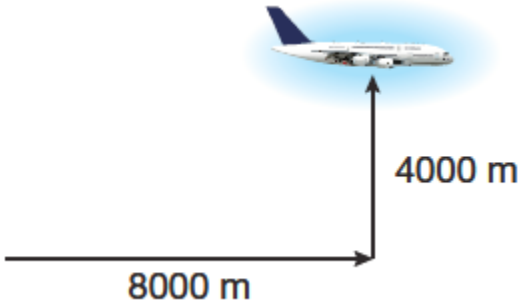
Add the two vectors using the head-to-tail method. The magnitude of the resultant vector can be found using the Pythagorean Theorem. Call the resultant vector $\vec{x}$.

$$|\langle 8000, 0\rangle|^2 + |\langle 0, 4000\rangle|^2 = |\vec{x}|^2$$

$$8000^2 + 4000^2 = |\vec{x}|^2$$

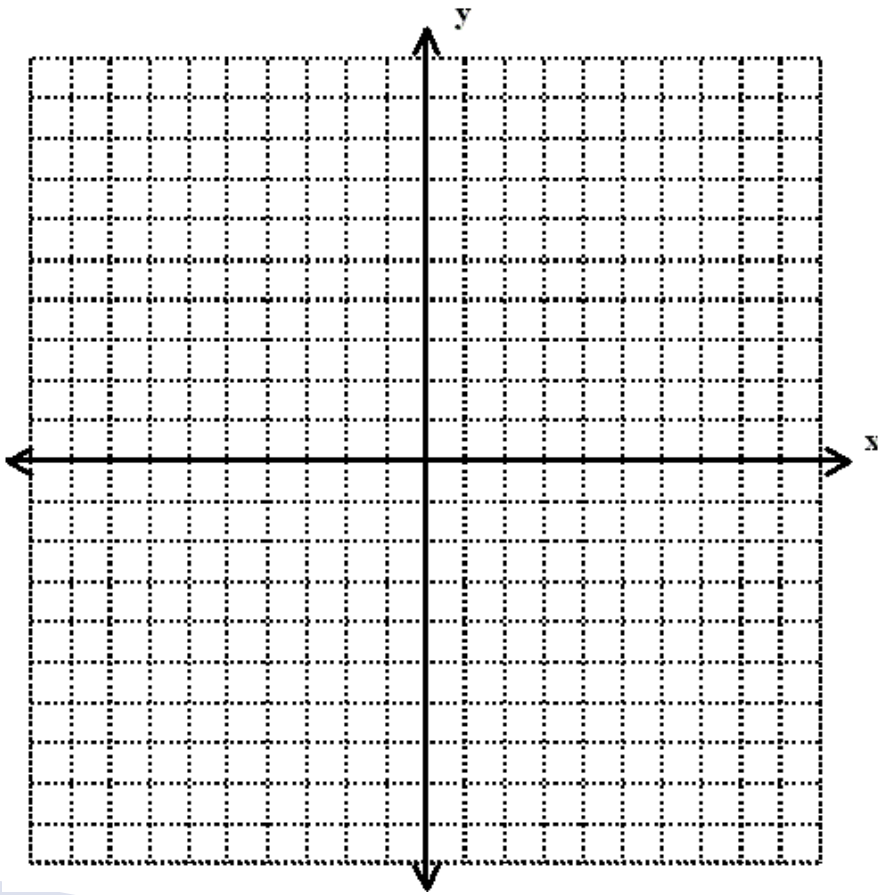
$$x \approx 8944$$

The plane has traveled approximately 8944 meters.



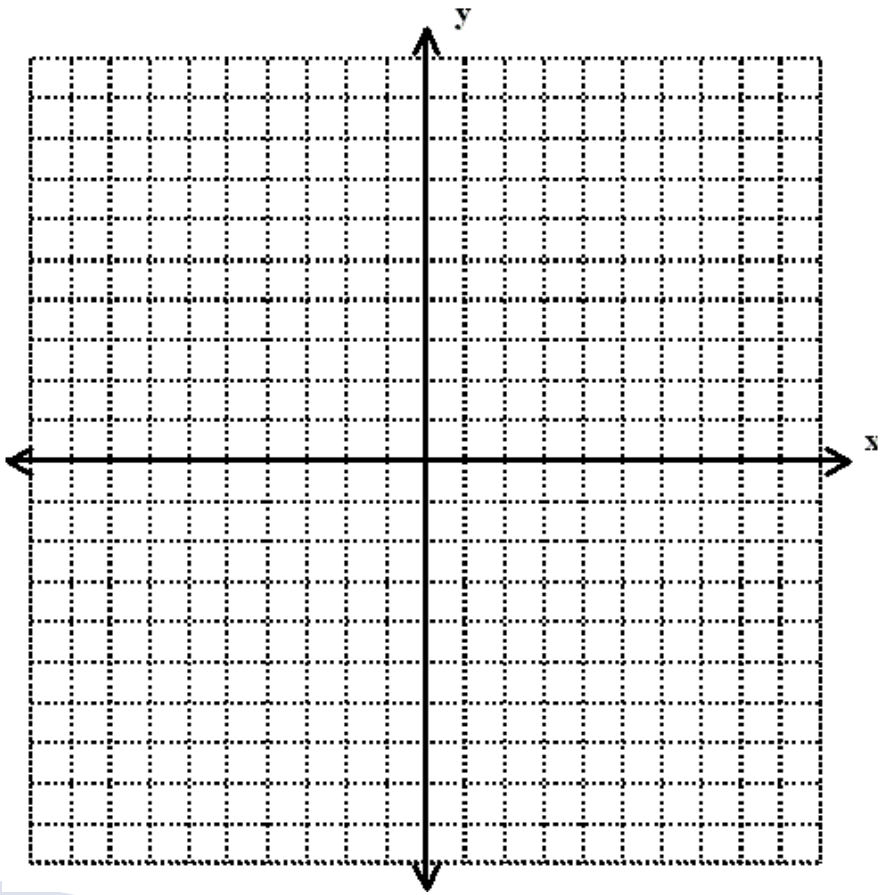
You Try!!!!!

a. Use the parallelogram method to add vectors $\vec{x} = \langle 3, 4 \rangle \; and \; \vec{y} = \langle 4, 1 \rangle$.



You Try!!!!!

b. Add the vectors $\vec{a} = \langle 1, 2 \rangle, \vec{b} = \langle 4, -5 \rangle, and\ \vec{c} = \langle -1, 7 \rangle$, using the head-to-tail method.



You Try!!!!!

c. Add the vectors $\vec{u} = \langle 3,0 \rangle \; and \; \vec{v} = \langle 0,-2 \rangle$. Use trigonometry to find the magnitude and direction of the resulting vector.

You Try!!!!!

d. A plane has traveled 2000 meters east while climbing to a height of 3000 meters. Write vectors to represent these two translations and find the magnitude of the resultant vector that gives the total distance the plane has traveled.

Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1-30 (Do the starred ones first)