Lesson 84

Dilations

A dilation is a transformation that changes the size of a figure but not its shape. The multiplier used on each dimension of a figure to change it into a similar figure is the scale factor.

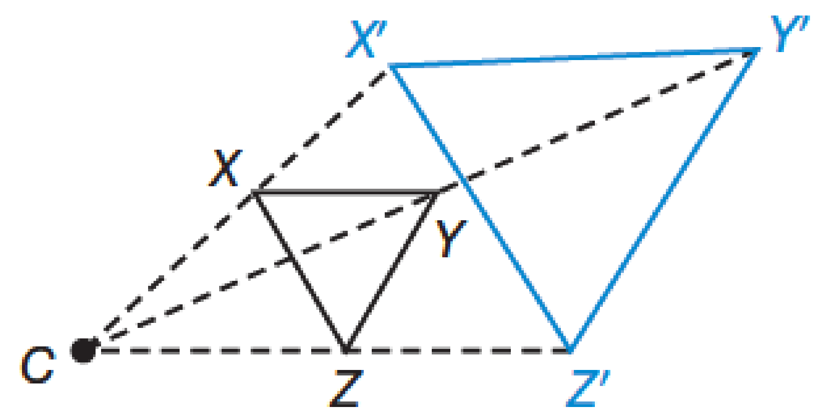
Dilations – A dilation maps a figure to a similar figure.

A dilation that results in an image smaller than its preimage is called a reduction or a contraction.

A dilation that results in an image larger than its preimage is called an enlargement or an expansion.

Dilations require a center and a scale factor. The center of dilation is the intersection of lines that connect each point of the image with the corresponding point of the preimage.

In the diagram, ΔXYZ a scale factor of 2, with the center of dilation C to create the image $\Delta X'Y'Z'$.



Hint

When a dilation is applied, it also affects the figure's distance from the center of dilation. For example, if a dilation of scale factor 2 is applied to a single point that is 3 units from the origin, the image will be 6 units from the origin.

Example 1 Enlarging by Dilation

Find the image of $\overline{AB}$ after a dilation with a scale factor of 2 and center *C*.

SOLUTION

The scale factor is greater than 1, so the dilation is an enlargement.

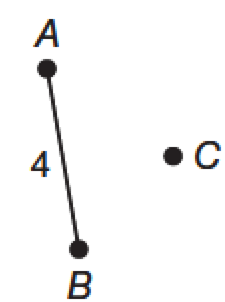
draw lines from the center *C* through the endpoints of the line segment.

Because the scale factor is 2, *CA'* = 2*CA* and *CB'* = 2*CB*.

Mark *A'* and *B'*.

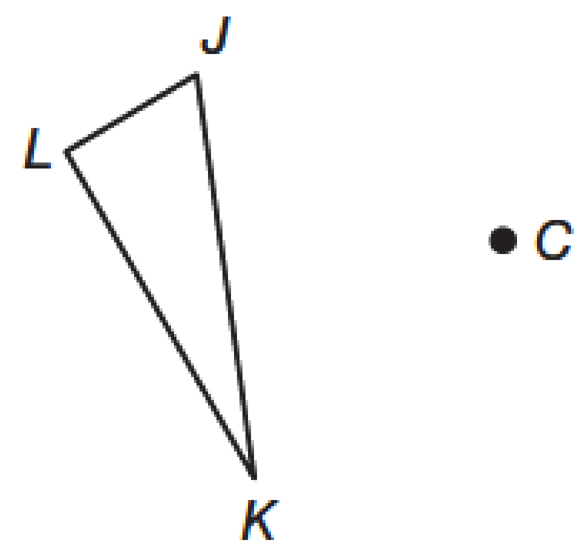
Connect *A'* and *B'* to form $\overline{A'B'}$.

Since *AB* = 4, and it was enlarged by a factor of 2, *A'B'* = 8.



Example 2 Contracting by Dilation

Apply a dilation to ΔJKL using a scale factor of $\frac{1}{2}$ and center *C.*
SOLUTION
This dilation is a reduction.
Draw lines from the center of dilation *C* to each of the vertices in ΔJKL.
Find the distance between *C* and each vertex.
Because the scale factor is $\frac{1}{2}$, $CJ' = \frac{1}{2}CJ, CL' = \frac{1}{2}CL, and\ CK' = \frac{1}{2}CK$.
Mark and label vertices *J'*, *K'*, and *L'*. Draw $\Delta J'K'L'$.



The scale factor can be used to find the coordinates of an image after a dilation. The notation $D_{O,k}$ indicates a dilation that is centered at the origin O of the coordinate plane and that has a scale factor of k. In mapping notation, $D_{O,k}\ (x, y) \to (kx, ky)$.

Example 3 Dilating on the Coordinate Plane

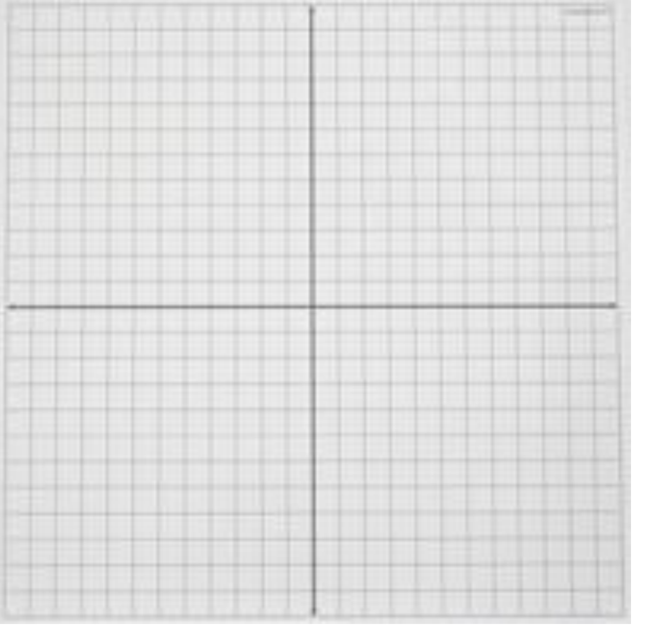
Triangle *DEF* has vertices located at *D*(4, 6), *E*(6, 2), and *F*(2, 4). Graph the image after a dilation centered at the origin and with a scale factor of $\frac{1}{2}$.

SOLUTION

Apply the transformation mapping given above.

$$D_{O,k}\,(x, y) \rightarrow (kx, ky)$$
$$D_{O,\frac{1}{2}}\,(4,6) \rightarrow (2,3)$$
$$D_{O,\frac{1}{2}}\,(6,2) \rightarrow (3,1)$$
$$D_{O,\frac{1}{2}}\,(2,4) \rightarrow (1,2)$$

Plot the points and draw $\Delta D'E'F'$ on the coordinate plane.



Example 4 Application: Photocopiers

A student wants to scan and enlarge a piece of art that is 6 inches by 8 inches. If the student selects the 150% enlargement function, what will the lengths of the sides of the copy be? How does the perimeter of the original art compare to the perimeter of the copy?

SOLUTION

An enlargement of 150% indicates the scale factor is 1.5.

The student should multiply each side of the original piece of art by 1.5.

The copy will be 9 inches by 12 inches.

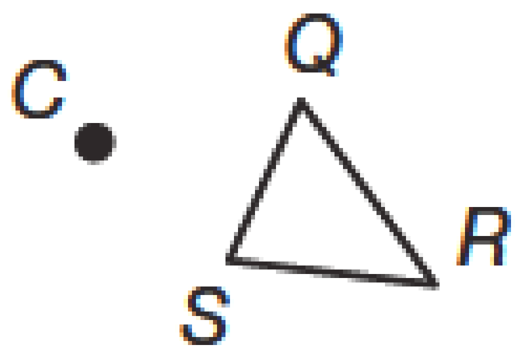
The perimeter of the original art is $6 + 6 + 8 + 8 = 28$ inches.

The perimeter of the copy is $9 + 9 + 12 + 12 = 42$ inches.

The original art has a perimeter that is $\frac{2}{3}$ the perimeter of the enlarged copy.

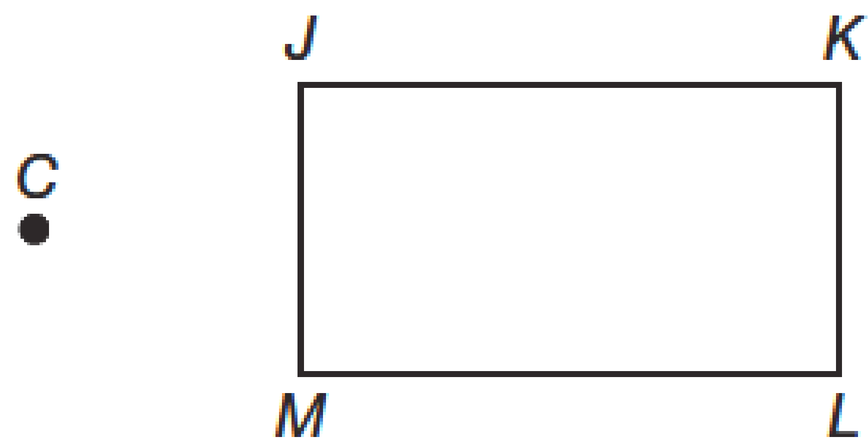
You Try!!!!

a. Apply a dilation to ΔQRS using a scale factor of 3 and center C.



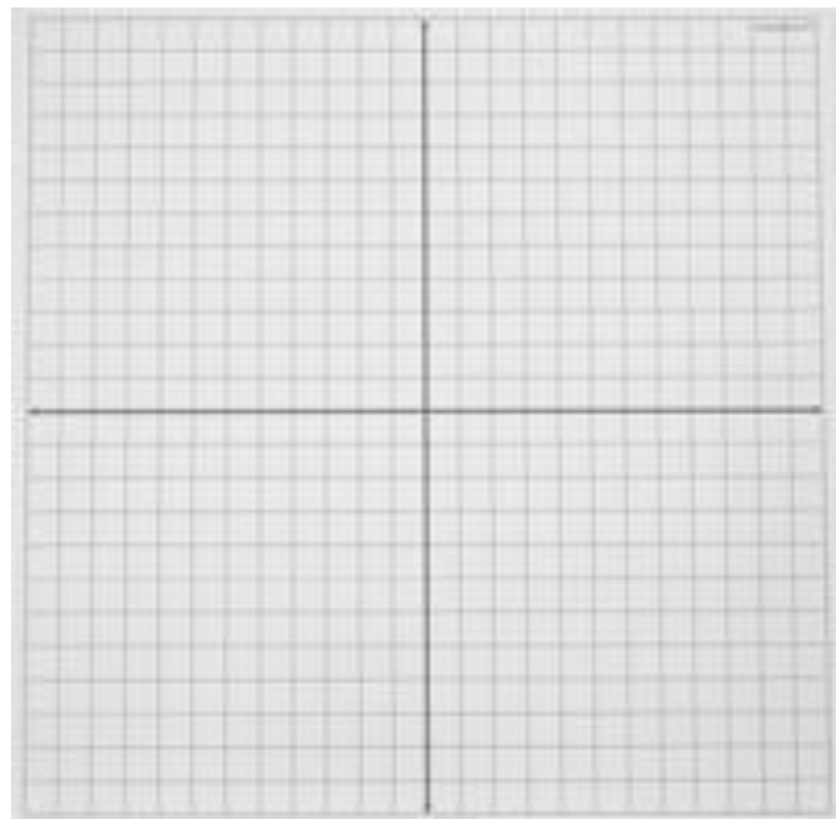
You Try!!!!

b. Apply a dilation to rectangle *JKLM* using a scale factor of $\frac{1}{2}$ and center *C*.



You Try!!!!

c. Triangle *MNP* has vertices *M*(−2, 1), *N*(−1, −2), and *P*(−3, −2). Apply a dilation with the center at the origin of the coordinate plane and a scale factor of 3.



You Try!!!!

- d.Architecture An architect is drawing plans for a building. The drawing for the front of the building is 4 feet long by 2.5 feet high. If the drawing is a reduction by a scale factor of $\frac{1}{20}$, what will the actual dimensions of the front of the building be? How do the areas of the drawing and the building compare?

Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)