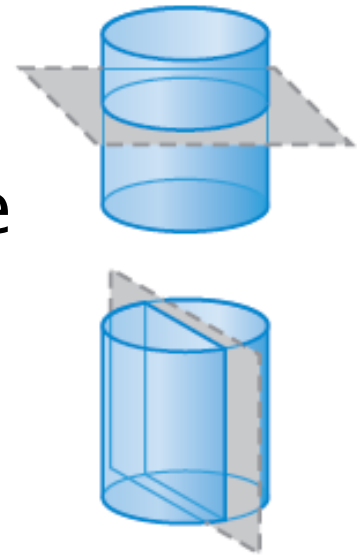


Lesson 85

Cross Sections of Solids

A cross section is the intersection of a three-dimensional figure and a plane. In the diagram, the first plane intersects the cylinder to make a circular cross section. The second plane intersects the cylinder to make a rectangle.



Think of a cross section as the shape that would be revealed if you cut straight through an object.

Example 1 Describing and Sketching Cross

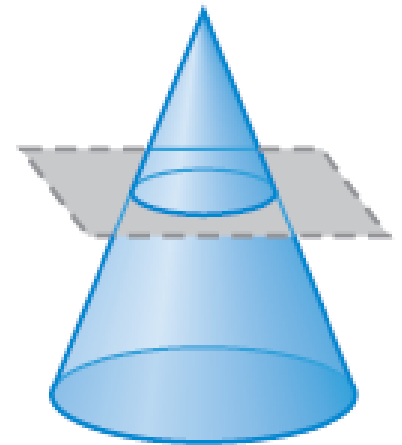
Sketching Cross

Describe and draw the cross section created by each plane.

a.

SOLUTION

The cross section is a circle.



Example 1 Describing and Sketching Cross

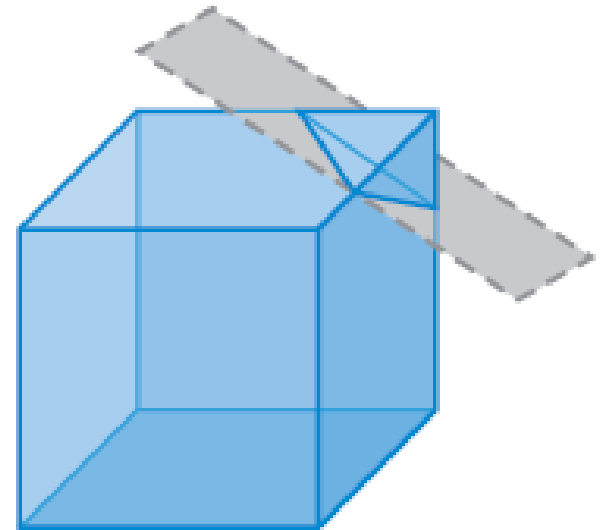
Sketching Cross

Describe and draw the cross section created by each plane.

b.

SOLUTION

The cross section is a triangle.



Example 2 Finding Perimeter of a Cross Section

a. If the plane shown is perpendicular to the altitude of the cylinder, what is the perimeter of the cross section?

SOLUTION

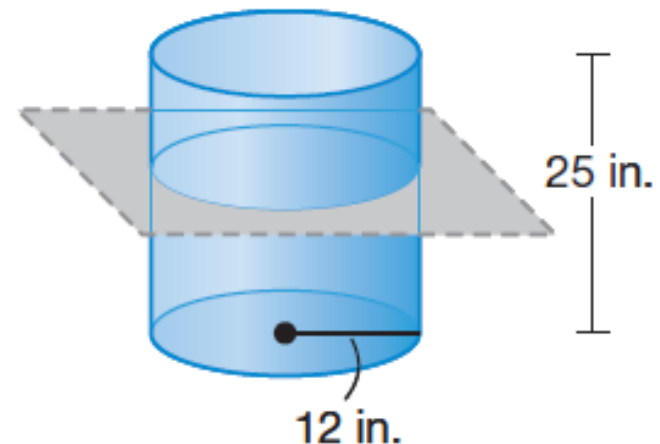
The cross section is a circle with the same radius as the cylinder itself.

Find the circumference of the circle in terms of π .

$$C = 2\pi r$$

$$C = 2\pi(12)$$

$$C = 24\pi \text{ inches}$$



Example 2 Finding Perimeter of a Cross Section

b. If the altitude of the cylinder lies on the plane shown, what is the perimeter of the cross section of the cylinder?

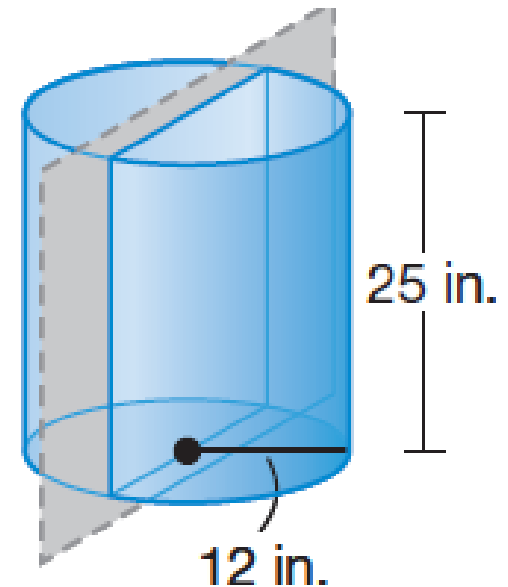
SOLUTION

The cross section is a rectangle. The height is 25 inches. Because the cross section travels through the center of the circle, the base of the rectangle is twice the radius.

$$P = 2b + 2h$$

$$P = 2(2)(12) + 2(25)$$

$$P = 98 \text{ inches}$$



Example 3 Finding Area of a Cross Section

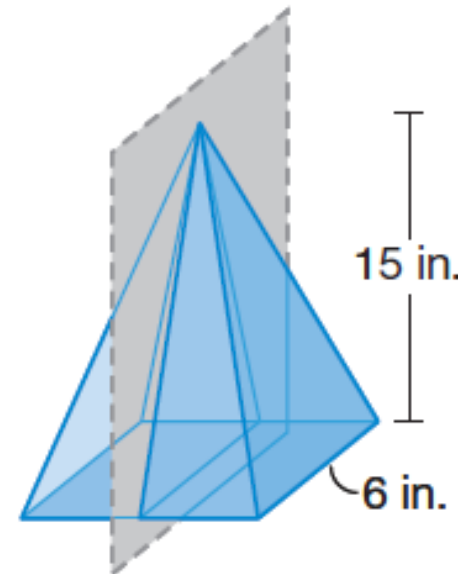
Find the area of this cross section of a square pyramid. The pyramid is 15 inches tall and the base is 6 inches wide. The cross section is perpendicular to the base of the pyramid and passes through the vertex.

SOLUTION

Draw the cross section with the appropriate lengths labeled. The cross section is a triangle with a height of 15 inches and a base of 6 inches.

Use the formula for area of a triangle.

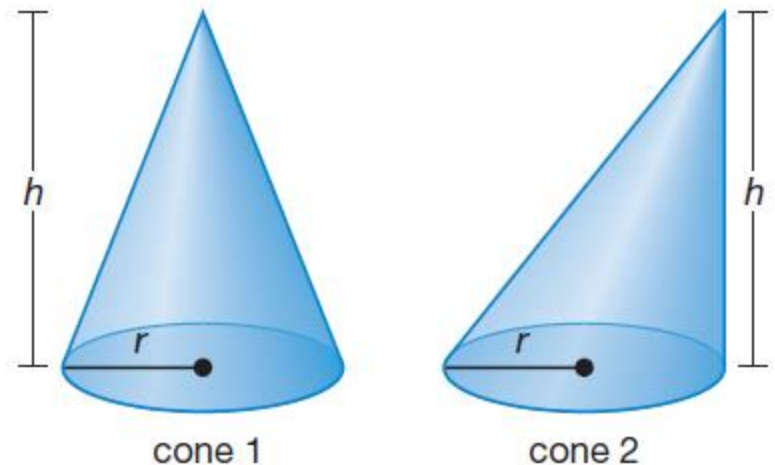
$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(15)(6)$$
$$A = 45 \text{ square inches}$$



Cavalieri's Principle – If two solids lying between parallel planes have equal heights and all cross sections at equal distances from their bases have equal areas, then the solids have equal volumes.

The two cones in the diagram illustrate Cavalieri's principle. The cones have the same radius and height, so Cavalieri's principle indicates that they will also have the same volume.

Imagine cutting the cones into many thin, circular cross sections. Each corresponding circular cross section of the two cones will be congruent.



Example 4 Application: Office Supplies

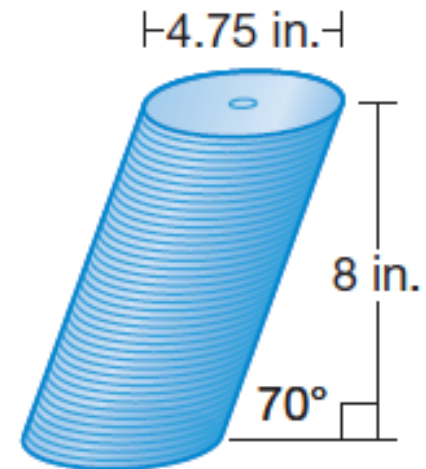
There is a stack of CDs on a desk. The stack is bumped and makes a 70° angle with the table. What is the volume of the stack? Is it equal to the volume of the stack if it made a 90° angle with the table?

SOLUTION

The stack has the same height before and after it is bumped. Moreover, each cross section represented by a CD has the same area.

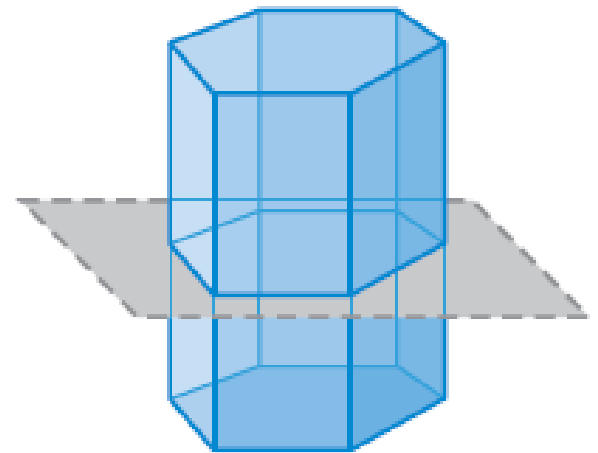
Therefore, according to Cavalieri's Principle, the cylinder has the same volume before it is bumped to the side as it does afterwards.

$$\begin{aligned}V &= \pi r^2 h \\V &= \pi \left(\frac{4.75}{2}\right)^2 (8) \\V &\approx 141.76 \text{ in}^3\end{aligned}$$



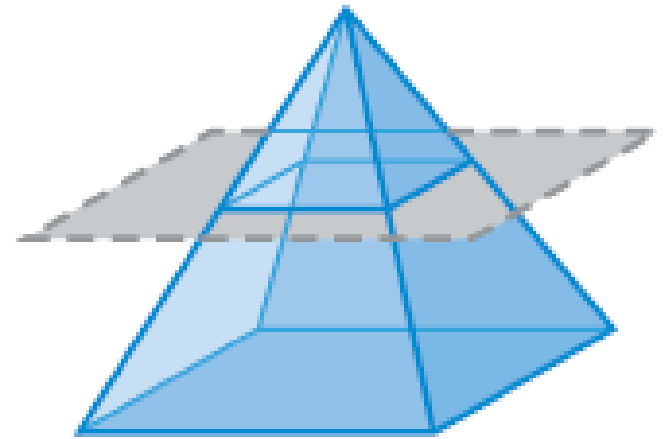
You Try!!!!

a. If the plane is parallel to the prism's bases, what is the shape of the cross section?



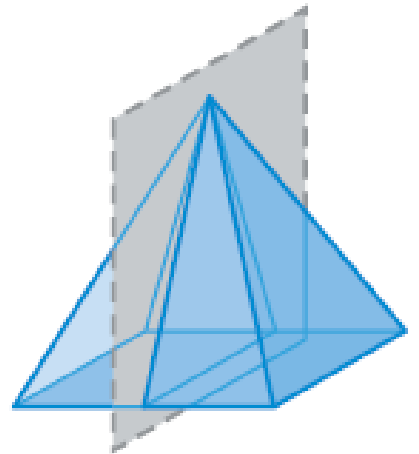
You Try!!!!

b. Find the perimeter of the cross section of the square pyramid if the cross section is parallel to the base. One side of the cross section is 4 inches long.



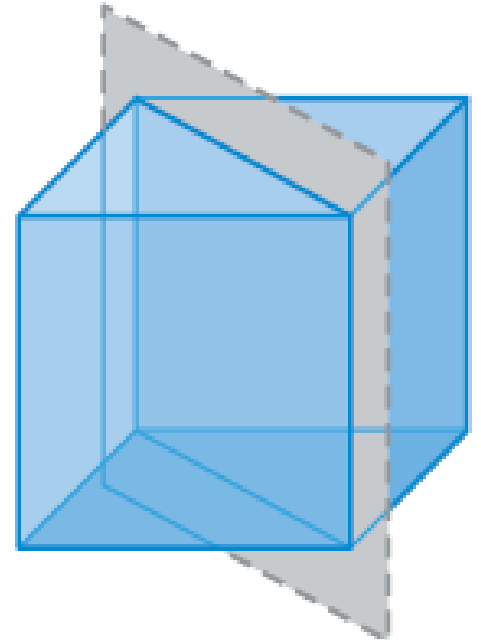
You Try!!!!

c. Find the perimeter of the cross section of the square pyramid. The cross section is perpendicular to the pyramid's base. The height of the pyramid is 14 inches and the base of the pyramid is 12 inches on each side. Round your answer to the nearest hundredth of an inch.



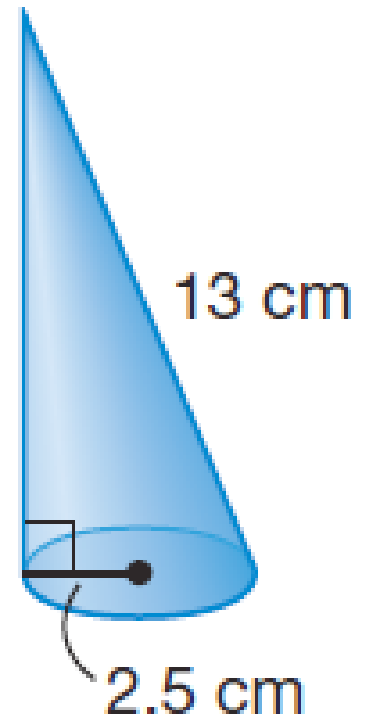
You Try!!!!

d. Find the area of the cross section. Each edge of the cube is 4 centimeters long. Round your answer to the nearest hundredth square centimeter.



You Try!!!!

e. Find the exact volume for the oblique cone.



Assignment

Page 555

Lesson Practice (Ask Mr. Heintz)

Page 556

Practice 1–30 (Do the starred ones first)