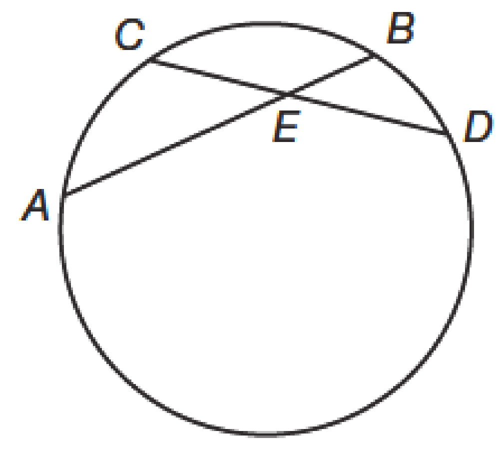
Lesson 86

Determining Chord Length

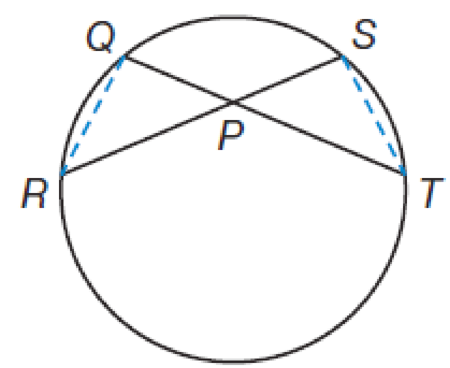
A chord is a segment whose endpoints lie on a circle. Theorem 86–1 relates the lengths of chord segments when two chords intersect.

Theorem 86–1 – If two chords intersect in a circle, then the products of the chord segments are equal. In the diagram, $(AE)(EB) = (CE)(ED)$.



Example 1 Proving Theorem 86-1

Given: Chords $\overline{TQ}$ and $\overline{RS}$ intersect at point *P*.
Prove: $(QP)(PT) = (RP)(PS)$
SOLUTION
Since two points determine a line, we can draw $\overline{QR}$ and $\overline{ST}$.
Because they intersect the same arc on the circle, $\angle RQT \cong \angle TSR$.
By the Vertical Angles Theorem, $\angle QPR \cong \angle SPT$.
Therefore, $\Delta QPR \sim \Delta SPT$ by the AA Similarity Postulate.
The corresponding sides of these similar triangles must be proportional, so $\frac{RP}{PT} = \frac{QP}{PS}$.
The cross product shows that $(QP)(PT) = (RP)(PS)$.



Example 2 Finding Chord Lengths

In the circle, chords $\overline{PQ}$ and $\overline{RS}$ intersect at T. Determine ST.

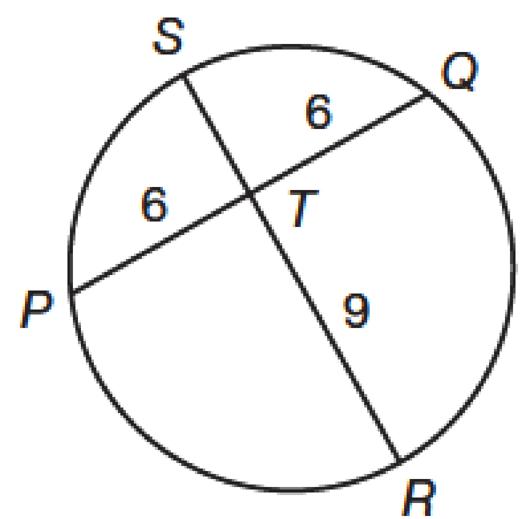
SOLUTION

Use Theorem 86–1 to write an expression relating the lengths of the chord segments.

$(PT)(QT) = (RT)(ST)$

$(6)(6) = (9)ST$

$ST = 4$



Example 3 Solving for Unknowns with Intersecting Chords

In this circle, use the expressions for the segment lengths to write and solve an equation for *x*.

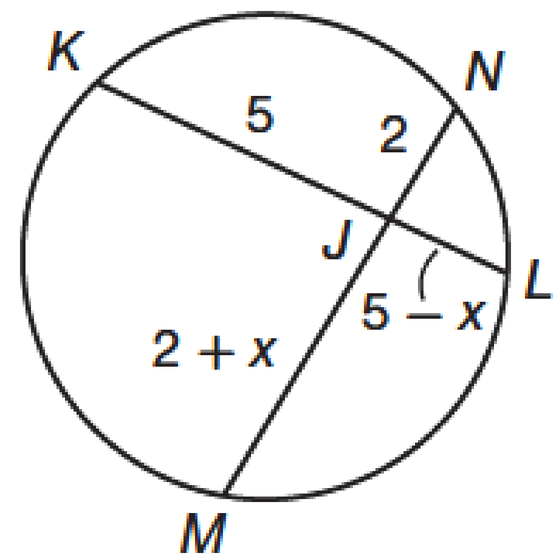
SOLUTION

$(JK)(JL) = (JM)(JN)$

$5(5 - x) = (2 + x)(2)$

$25 - 5x = 4 + 2x$

$x = 3$



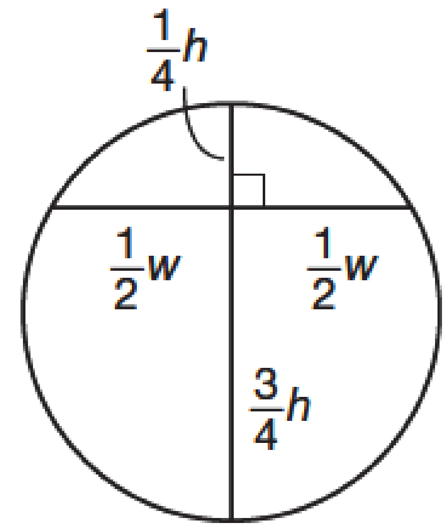
Example 4 Application: Aviation

A "super-heavy" passenger jet has an upper passenger deck that is located $\frac{3}{4}$ of the way up the cylindrical fuselage. What percentage of the height of the fuselage is the width of the upper deck?

SOLUTION

Understand: Draw a diagram. A cross-section of the fuselage is circular, as shown.

Plan: Use Theorem 86-1 to write an equation.



$$\left(\frac{1}{2}w\right)\left(\frac{1}{2}w\right)=\left(\frac{1}{4}h\right)\left(\frac{3}{4}h\right)$$

$$\frac{1}{4}w^2=\frac{3}{16}h^2$$

$$\left(\frac{4}{1}\right)\cdot\frac{1}{4}w^2=\frac{3}{16}h^2\cdot\left(\frac{4}{1}\right)$$

$$w^2=\frac{3}{4}h^2$$

$$\sqrt{w^2}=\sqrt{\frac{3}{4}h^2}$$

$$w=\frac{\sqrt{3}}{2}h$$

$$w\approx .87h$$

The width of the upper deck is approximately 0.87, or 87% of the height of the fuselage.

Check: Look at the diagram. Does it look like the upper deck is a little shorter than, close to the same length as, the height of the fuselage? It appears to be that way, so the answer seems correct.

You Try!!!!

In $\odot G$, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. Use this information for parts a and b.

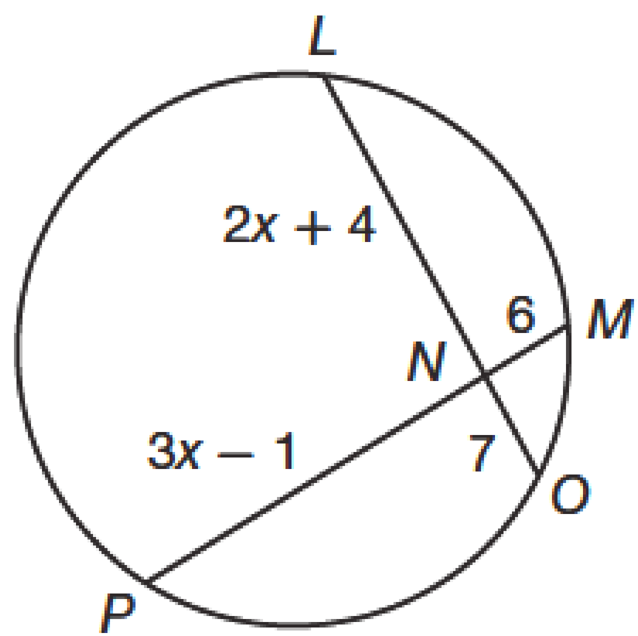
a. Determine DE if $AE = 3$, $BE = 16$, and $CE = 9$.

b. Suppose $AE = 7$, $BE = y$, $CE = 4 - y$, and $DE = 2$.

Write and solve an equation for y.

You Try!!!!

c. In the diagram, $\overline{LO}$ and $\overline{PM}$ intersect at *N*. Find the value of *x*.



You Try!!!!

d.Civil Engineering A cylindrical natural gas pipeline is supported at two points that are 10% of the diameter of the pipeline above its lowest point. If the diameter of the pipeline is 4 feet, 9 inches, how far apart are the supports?

Assignment

Page 562

Lesson Practice (Ask Mr. Heintz)

Page 562

Practice 1–30 (Do the starred ones first)