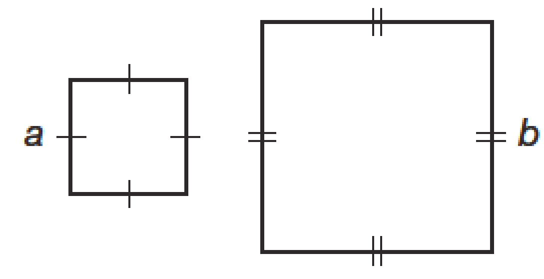
Lesson 87

Area Ratios of Similar Figures

Recall that polygons are similar if they have the same shape, but differ in size. This difference in size describes their scale factor to each other and can be written as a similarity ratio.

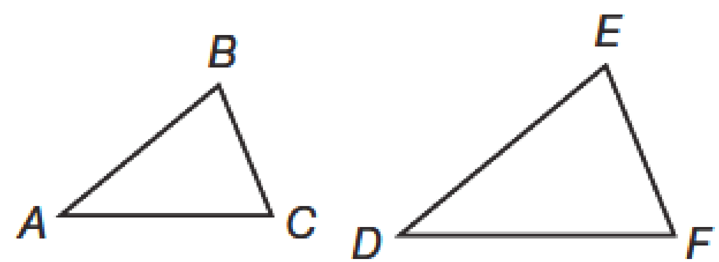
For the squares given, the perimeter of the first square is $4a$ and the second is $4b$. The ratio of their perimeters is $4a{:}4b$, which can be reduced to $a{:}b$. Their areas are a^2 and b^2, so the ratio of their areas is $a^2{:}b^2$. These relationships are true of all similar polygons.



Theorem 87-1 – If two similar figures have a scale factor of a:b, then the ratio of their perimeters is a:b, and the ratio of their areas is $\boldsymbol{a^2}:\boldsymbol{b^2}$.

Example 1 Proving Theorem 87-1

Prove the first part of Theorem 87-1.
Given: $\Delta ABC \sim \Delta DEF$
Prove: $\frac{AB+BC+AC}{DE+EF+DF} = \frac{AB}{DE}$
SOLUTION



Let x be the similarity ratio of $AB:DE$.
In other words, $x = \frac{AB}{DE}$
Then $AB = (DE)(x)$.
Furthermore, $BC = (EF)(x)$ and $CA = (FD)(x)$.
By the Addition Property of Equality,
$AB + BC + CA = (DE)(x) + (EF)(x) + (FD)(x)$.
Therefore, $AB + BC + CA = x(DE + EF + FD)$. By the Division Property of Equality, $x = \frac{AB+BC+AC}{DE+EF+DF}$.
By substitution, $\frac{AB+BC+AC}{DE+EF+DF} = \frac{AB}{DE}$

Example 2 Ratio of Perimeters of Similar Figures

In the given similar figures, the perimeter of the smaller shape is 50 inches. Determine the perimeter of the larger shape.

SOLUTION

The similarity ratio of the larger rectangle to the smaller rectangle is $12x:4x$, or 3:1 after being reduced.

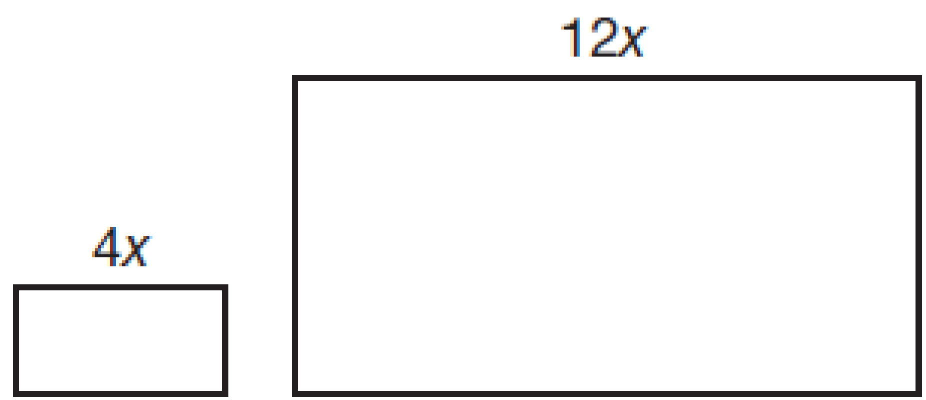
By Theorem 87-1, the perimeters of the two rectangles will be in the same ratio.

Set up a proportion to solve for the larger figure's perimeter.

$$\frac{3}{1} = \frac{P}{50}$$

$$P = 150$$

So the perimeter of the larger rectangle is 150 inches.



Example 3 Ratio of Areas of Similar Figures

The two triangles given have a similarity ratio of 2:5. Determine the ratio of their areas and the area of the smaller triangle.

SOLUTION

From Theorem 87-1, the ratio of their areas will be $a\,2 : b\,2$.

Therefore, $a^2 : b^2$ the ratio of the smaller triangle's area to the larger triangle's area is 4:25.

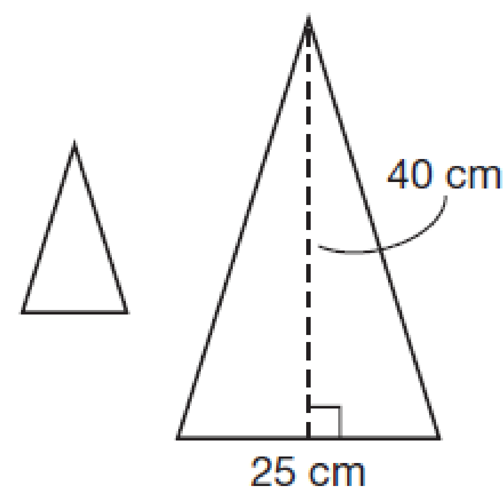
To find the area of the smaller triangle, first find the area of the larger triangle.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(25)(40)$$
$$A = 500$$

Now set up a proportion using the ratio of the triangles' areas to find the area of the small triangle.

$$\frac{4}{25} = \frac{A}{500}$$
$$A = 80$$

The area of the smaller triangle is 80 square centimeters.



Example 4 Application: Landscape Design

A landscape design company has created a plan for a large garden in the shape of an isosceles trapezoid, as illustrated in the diagram. The diagram of the garden is in a 2:355 ratio with the size of the actual garden. Find the perimeter and area of the actual garden.

SOLUTION

Add the sides together to find the perimeter of the shape.

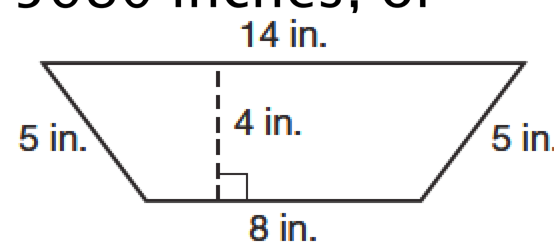
$$P_1 = 14 + 5 + 8 + 5 = 32$$

The perimeter of the scale garden is 32 inches. Write a proportion to find the perimeter of the real garden.

$$\frac{2}{35} = \frac{32}{P^2}$$

$$P^2 = 5680$$

The perimeter of the actual garden will be 5680 inches, or approximately 473 feet.



Example 4 Application: Landscape Design

Apply the formula for area of a trapezoid.

$$A_1 = \left(\frac{b_1 + b_2}{2}\right)h$$

$$A_1 = \left(\frac{14 + 8}{2}\right)4$$

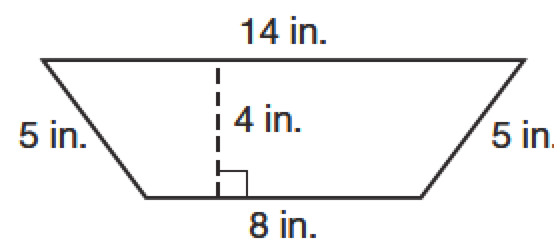
$$A_1 = 44$$

Therefore, the area of the trapezoid in the diagram is 44 square inches. The area ratio will be given by $2^2 : 355^2$, which is 4:126,025. Applying this ratio, we obtain

$$\frac{4}{126025} = \frac{44}{A_2}$$

$$A_2 = 1386275$$

The area of the garden is 1,386,275 square inches or approximately 9627 square feet.

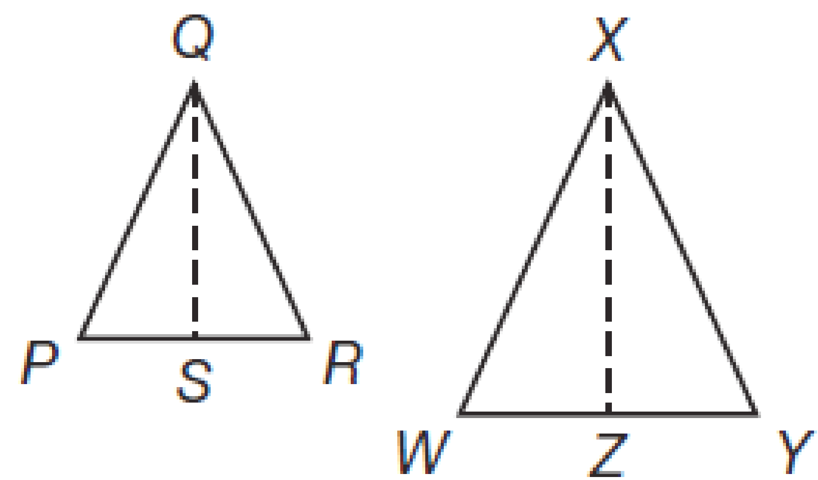


You Try!!!!

a.Prove the second part of Theorem 87-1.

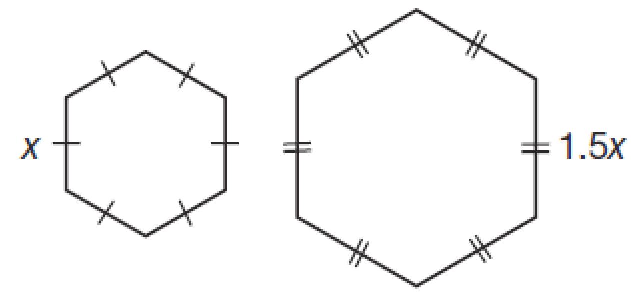
Given: $\Delta PQR \sim \Delta WXY$

Prove: $\frac{AREA\ \Delta PQR}{AREA\ \Delta WXY} = \frac{PR^2}{WY^2}$

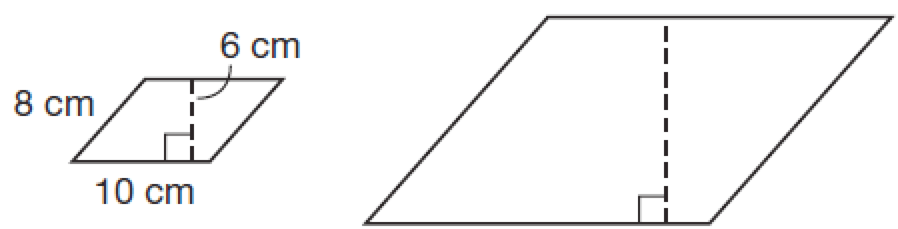


You Try!!!!

b.In the given similar figures, the perimeter of the large hexagon is 120 feet. Determine the perimeter of the small hexagon.



c.The two parallelograms given have a similarity ratio of 2:5. Determine the ratio of their areas and the area of the larger parallelogram.



You Try!!!!

d.The kitchen on a floor plan shows a triangle from the sink to the refrigerator to the counter that has an area of 1.5 square feet. If the floor plan has a scale of 1:10, what will be the actual area of this triangle when the house is built?

Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)