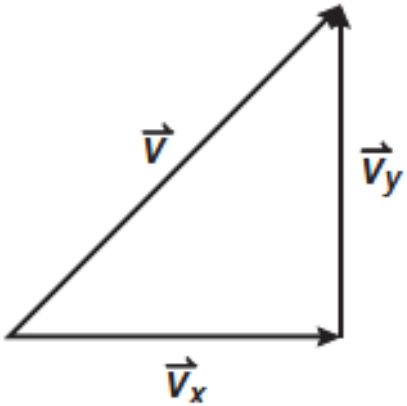
Lesson 89

Vector Decomposition

Decomposing a vector means to separate it into two vectors which, when added, result in the original vector. When a vector is decomposed, it is split into a horizontal vector and a vertical vector. In the diagram, $\vec{v}$ is decomposed into vertical, $\overrightarrow{v_y}$ and horizontal, $\overrightarrow{v_x}$.

Notice that the vector makes a right triangle with its vertical and horizontal components. Since it makes a right triangle, trigonometric ratios can be used to find the magnitude of the vertical and horizontal components.

The angle of a vector is always given counterclockwise from the positive x–axis.



Example 1 Decomposing Vectors

Decompose each vector. Round your answer to the nearest hundredth.

a. $\langle 2, 3 \rangle$

SOLUTION

A vector in component form is easily decomposed. The *x* and *y* components of the vector are given.

$\overrightarrow{v_x} = \langle 2,0 \rangle$ *and* $\overrightarrow{v_y} = \langle 0,3 \rangle$

Example 1 Decomposing Vectors

Decompose each vector. Round your answer to the nearest hundredth.

b. $\vec{v}$ has a 33° angle and a magnitude of 7.

SOLUTION

Draw the vector. Use trigonometry to find the magnitudes of $\overrightarrow{v_x}$ *and* $\overrightarrow{v_y}$.

$$\cos 33° = \frac{\left|\overrightarrow{v_x}\right|}{7}$$

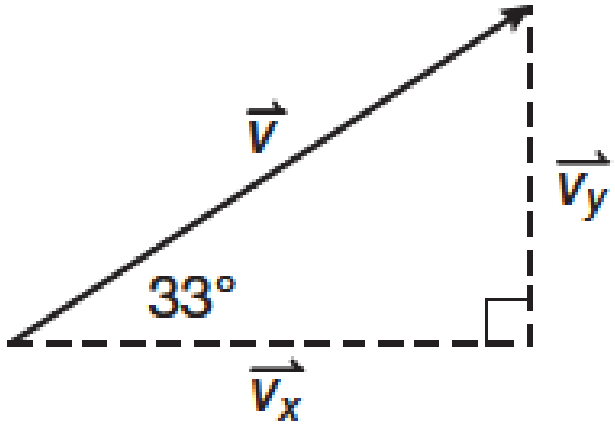
$$7\cos 33° = \left|\overrightarrow{v_x}\right|$$

$$\left|\overrightarrow{v_x}\right| \approx 5.87$$

$$\sin 33° = \frac{\left|\overrightarrow{v_y}\right|}{7}$$

$$7\sin 33° = \left|\overrightarrow{v_y}\right|$$

$$\left|\overrightarrow{v_y}\right| \approx 3.81$$



$\overrightarrow{v_x}$ is a horizontal vector, its component form is $\langle 5.87, 0\rangle$ *and*, and the component form of the vertical vector $\overrightarrow{v_y}$ is ⟨0, 3.81⟩.

Vectors can also be added by decomposition. To add two vectors, decompose them and add their vertical and horizontal components. This will give the resultant vector in component form.

Example 2 Adding Vectors by Decomposition

Vector $\vec{v}$ has a magnitude of 4 and makes a 25° angle with the horizontal.
Vector $\vec{w}$ has a magnitude of 7 and makes a 61° angle with the horizontal.
Add the vectors by decomposition. Find the magnitude and the angle the resultant vector makes with the horizontal to the nearest tenth.

SOLUTION

First, decompose $\vec{v}$.

$$\cos 25° = \frac{\left|\overrightarrow{v_x}\right|}{4}$$

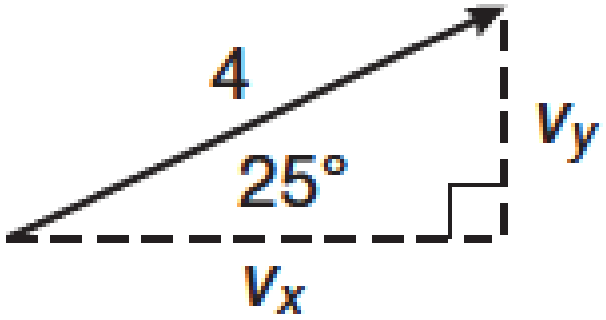
$$4\cos 25° = \left|\overrightarrow{v_x}\right|$$

$$\left|\overrightarrow{v_x}\right| \approx 3.63$$

$$\sin 25° = \frac{\left|\overrightarrow{v_y}\right|}{4}$$

$$4\sin 25° = \left|\overrightarrow{v_y}\right|$$

$$\left|\overrightarrow{v_y}\right| \approx 1.69$$



Example 2 Adding Vectors by Decomposition

Then, decompose $\vec{w}$.

$$\cos 61° = \frac{\left|\overrightarrow{w_x}\right|}{7}$$

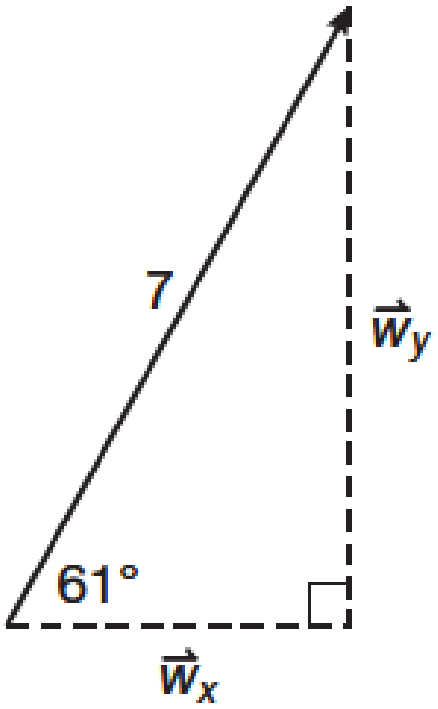
$$7\cos 61° = \left|\overrightarrow{w_x}\right|$$

$$\left|\overrightarrow{w_x}\right| \approx 3.39$$

$$\sin 61° = \frac{\left|\overrightarrow{w_y}\right|}{7}$$

$$7\sin 61° = \left|\overrightarrow{w_y}\right|$$

$$\left|\overrightarrow{w_y}\right| \approx 6.12$$



Example 2 Adding Vectors by Decomposition

Next, add the magnitudes of the components.

$$|\overrightarrow{v_x}| + |\overrightarrow{w_x}| \approx 3.63 + 3.39 \approx 7.02$$
$$|\overrightarrow{v_y}| + |\overrightarrow{w_y}| \approx 1.69 + 6.12 \approx 7.81$$

Now that we know the magnitudes of the legs of the right triangle, we can use the Pythagorean Theorem to find the magnitude of the resultant vector.

$$a^2 + b^2 = c^2$$
$$(7.02)^2 + (7.81)^2 \approx c^2$$
$$\sqrt{110.28} \approx c$$
$$c \approx 10.5$$

Example 2 Adding Vectors by Decomposition

Finally, use one of the inverse trigonometric functions to find the angle measure that the resultant vector makes with the horizontal.

Because you know the values of all three sides, you can use any function.

$$\theta = tan^{-1}\frac{7.81}{7.02}$$

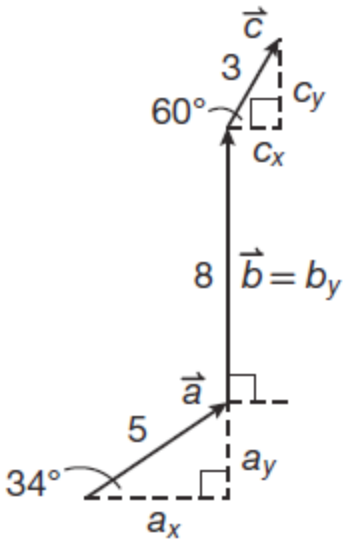
$$\theta \approx 48°$$

The resultant vector has a magnitude of approximately 10.50 and makes a 48° angle with the horizontal.

Example 3 Adding More Than Two Vectors by Decomposition

Vector $\vec{a}$ has a magnitude of 5 and makes a 34° angle counterclockwise.
Vector $\vec{b}$ has a magnitude of 8 and makes a 90° angle counterclockwise.
Vector $\vec{c}$ has a magnitude of 3 and makes a 60° angle counterclockwise.
Add the vectors by decomposition. Find the magnitude and angle of the resultant vector.
SOLUTION
Draw the vectors head to tail. Use the drawing to estimate the magnitude and angle of the resultant vector.
First, decompose each vector.

$$\cos 34° = \frac{\left|\overrightarrow{a_x}\right|}{5}$$
$$5\cos 34° = \left|\overrightarrow{a_x}\right|$$
$$\left|\overrightarrow{a_x}\right| \approx 4.15$$
$$\sin 34° = \frac{\left|\overrightarrow{a_y}\right|}{5}$$
$$5\sin 34° = \left|\overrightarrow{a_y}\right|$$
$$\left|\overrightarrow{a_y}\right| \approx 2.80$$



Example 3 Adding More Than Two Vectors by Decomposition

Because $\vec{b}$ has no horizontal component,

$$\left|\overrightarrow{b_x}\right| = 0$$
$$\left|\overrightarrow{b_y}\right| = 8$$

$$\cos 60° = \frac{\left|\overrightarrow{c_x}\right|}{3}$$
$$3\cos 60° = \left|\overrightarrow{c_x}\right|$$
$$\left|\overrightarrow{c_x}\right| \approx 1.5$$
$$\sin 60° = \frac{\left|\overrightarrow{c_y}\right|}{3}$$
$$3\sin 60° = \left|\overrightarrow{c_y}\right|$$
$$\left|\overrightarrow{c_y}\right| \approx 2.60$$

Example 3 Adding More Than Two Vectors by Decomposition

Add the magnitudes of the horizontal and vertical vectors.

$$|\overrightarrow{a_x}| + |\overrightarrow{b_x}| + |\overrightarrow{c_x}| \approx 4.15 + 0 + 1.5 \approx 5.65$$

$$|\overrightarrow{a_y}| + |\overrightarrow{b_y}| + |\overrightarrow{c_y}| \approx 2.80 + 8 + 2.60 \approx 13.40$$

Next, use the Pythagorean Theorem to find the magnitude of the resultant vector.

$$a^2 + b^2 = c^2$$

$$(5.65)^2 + (13.40)^2 \approx c^2$$

$$\sqrt{211.48} \approx c$$

$$c \approx 14.54$$

Finally, find the angle θ that the resultant vector makes with the horizontal.

$$\theta = sin^{-1}\frac{13.40}{14.54}$$

$$\theta \approx 67°$$

Example 4 Application: Camping

You can use vectors to calculate forces acting on an object. For example, if a tent pole is placed at a 58° angle and is supporting a load of 15 pounds, what are the vertical and horizontal loads on the pole? Round your answers to the nearest hundredth of a pound.

SOLUTION

The magnitude of the vector is 15 pounds.

Use the sine function to find the vertical component.

$$\sin 58° = \frac{y}{15}$$

$$y \approx 12.72$$

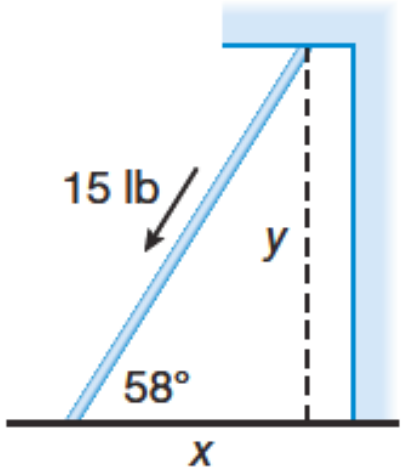
The vertical load is about 12.72 lbs.

Use the cosine function to find the horizontal component.

$$\cos 58° = \frac{x}{15}$$

$$x \approx 7.95$$

The horizontal load is about 7.95 lbs.



You Try!!!!

Round to the nearest hundredth or nearest degree.

a.Vector $\vec{d}$ has a magnitude of 100 and makes a 55° angle with the horizontal. Decompose the vector into its *x*- and *y*-components.

You Try!!!!

b.Vector $\vec{p}$ makes a 35° angle with the horizontal. Vector $\vec{q}$ makes a 40° angle with the horizontal. If $\vec{p}$ has a magnitude of 30 and $\vec{q}$ has a magnitude of 40, what is the angle and magnitude of the resultant vector when $\vec{p}$ and $\vec{q}$ are added?

You Try!!!!

c. Vector $\vec{t}$ has a magnitude of 12 and a direction of 80°. Vector $\vec{u}$ has a magnitude of 16 and a direction of 0°. Vector $\vec{v}$ has a magnitude of 8 and a direction of 64°. Find the magnitude and angle of their sum.

You Try!!!!

d.Camping The pole for a tent makes an 86° angle with the ground. If the load on the pole is 25 pounds, what are the vertical and horizontal loads?

Assignment

Page 582

Lesson Practice (Ask Mr. Heintz)

Page 582

Practice 1-30 (Do the starred ones first)