# Lesson 90

## Composite Transformations

Composite Transformation – A combination of transformations.

If a figure is both translated 3 units to the right and rotated 90°, for example, it has undergone a composite transformation.

Composition of Two Isometries - The composition of two isometries is an isometry.

Recall that translations, rotations, and reflections are all isometries. A composite transformation with two or more of these transformations will also be an isometry.

A glide reflection is a composition of a translation and a reflection across a line parallel to the translation vector. Since a glide reflection combines translation and reflection, it is an isometry.

# Example 1 Performing a Glide Reflection

Reflect $\Delta ABC$ across line $m$ and then translate it along $\vec{v}$.

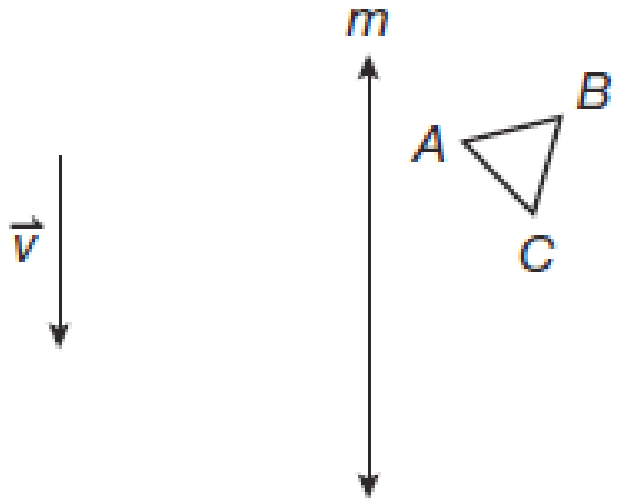
SOLUTION

Draw the reflection of $\Delta ABC$. The vector $\vec{v}$ indicates a translation downwards. Shift the image $\Delta A'B'C'$ down to complete the glide reflection.

After the composite transformation, the new image is $\Delta A''B''C''$.

**Math Reasoning**

**Verify** In Example 1, what would be the result of performing the translation first, followed by the reflection?



Two Reflections Across Two Parallel Lines – The composition of two reflections across two parallel lines is a translation.

If a figure is reflected twice, the end result is an image that represents a translation from the original preimage. Example 2 demonstrates this.

# Example 2 Translating by Composite Reflection

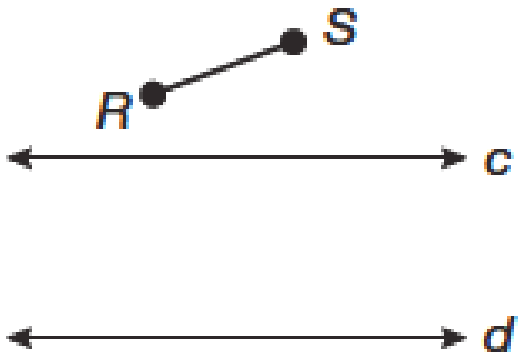
Reflect the line segment across *c*, and then reflect the image across *d*. In the diagram, $c \parallel d$.

SOLUTION

Draw the first reflection across line *c*. The result is a segment with a slope that is the reciprocal of the preimage's slope.

Draw the second reflection across line *d*.

The result is the original preimage, translated down.



Two Reflections Across Two Intersecting Lines – The composition of two reflections across two intersecting lines is a rotation about the point of intersection. The angle of rotation is twice the angle formed by the intersecting lines.

If a figure is reflected across lines that intersect, it results in a rotation of the figure. Example 3 demonstrates this.

# Example 3 Rotating by Composite Reflection

Reflect rectangle $STUV$ across $\overleftrightarrow{JK}$, and then reflect the image across $\overleftrightarrow{LM}$.

SOLUTION

The diagram shows two images. The rectangle $S'T'U'V'$ is the first reflection across $\overleftrightarrow{JK}$.

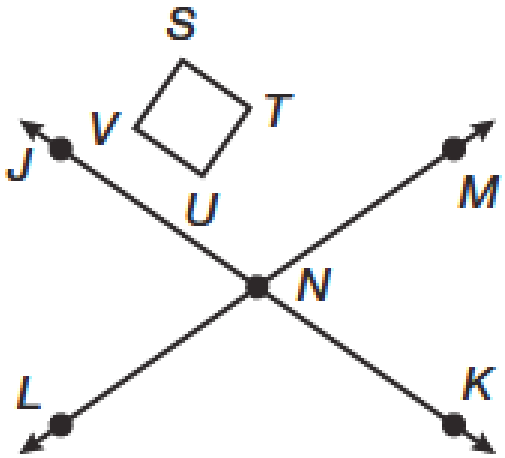
Reflecting this image across $\overleftrightarrow{LM}$ results in $S''T''U''V''$.

Notice that the final image is just a rotation of $STUV$ around $N$, the intersection of the two lines.

Applying the same transformations to a figure in a different order sometimes results in a different image.

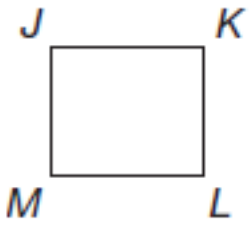
**Math Reasoning**

**Analyze** If you continue to reflect $STUV$ clockwise across $\overleftrightarrow{JK}$ and $\overleftrightarrow{LM}$, what will the final result be?



# Example 4 Analyzing Order of Composition

a. Perform transformations on the rectangle in the following order: reflection across a vertical line, rotation 90° clockwise, rotation 90° clockwise, and reflection across a horizontal line. What is the result?




**Math Reasoning**

**Analyze** Write one way to perform the composition in this example in a different order and get the same final image as the one in **a.**

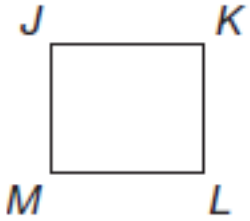
SOLUTION

The four transformations returned the rectangle to its original orientation.

Such a series of transformations is called an identity transformation.

# Example 4 Analyzing Order of Composition

b. Perform transformations on the original rectangle in part a in the following order: reflection across a vertical line, rotation 90° clockwise, reflection across a horizontal line, and rotation 90° clockwise. What is the result?




SOLUTION

The four transformations resulted in an image that is not oriented the same as the original preimage, so this is not an identity transformation.

# Example 4 Analyzing Order of Composition

c. Compare the final images in parts a and b. What do the results indicate?

SOLUTION

The final images in parts a and b are oriented differently.

This indicates that the order in which the transformations are performed affects the final result.

# Exploration

In this exploration, you will perform a composite transformation beginning with a given preimage. You will compare your final image to those of your classmates and to a final image provided by your teacher.

1. On grid paper, create $\Delta XYZ$ with $X(0,0)$, $Y(3,0)$, and $Z(0,4)$.
2. Follow the series of translations described by your teacher to generate an image based on composite transformations.
3. Redraw $\Delta XYZ$ on a new grid and follow a second series of translations described by your teacher to generate an image.
4. Compare your results for the two composite transformations with a classmate.
5. Check your results with the results that your teacher posts.

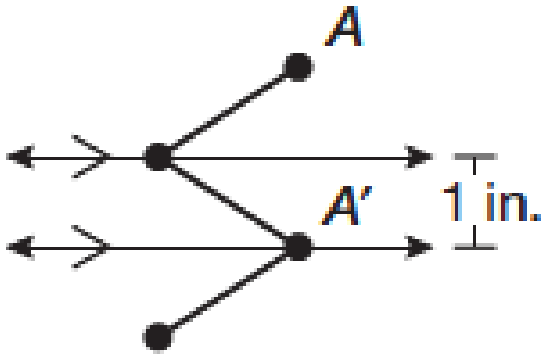
# Example 5 Application: Design

To create a design, Reina draws a line segment, and reflects it twice as shown. The reflection lines are parallel and are 1 inch apart. Find the distance from $A$ to $A'$.

SOLUTION

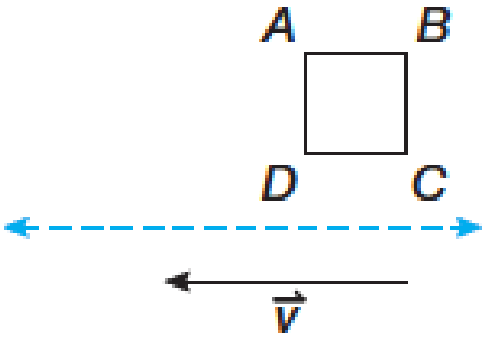
Since the reflection creates an isometry, the distance from point $A'$ on the lower reflection line to the upper reflection line is the same as the distance from the upper reflection line to point $A$.

Therefore, the distance from $A$ to $A'$ is 2 inches.

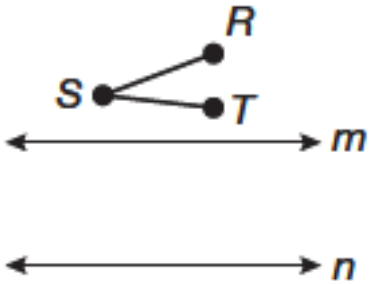


# You Try!!!!!

a. Reflect the square over the dashed line and translate it by $\vec{v}$.

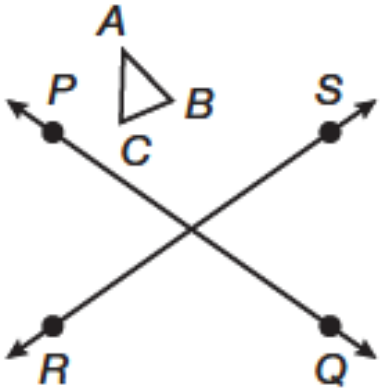



b. Reflect the figure across *m*, and then reflect the image across *n*.

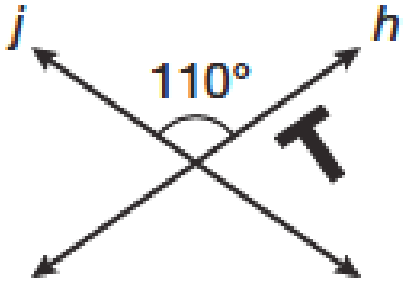


# You Try!!!!!

c. Reflect $\Delta ABC$ across $\overleftrightarrow{PQ}$, and then reflect the image across $\overleftrightarrow{RS}$ .




e. To create a new logo for a sweatshirt, a designer reflects the letter *T* across line *h*. This image is then reflected across line *j*. What single transformation could move the letter *T* from its starting point to its final position?



# Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1-30 (Do the starred ones first)