Name: _____

Date: _____

Period: _____

Geometry Lesson 90

Objective: TSW identify and use composite transformations.

Composite Transformation - A _____ of transformations.

If a figure is both translated 3 units to the right and rotated 90°, for example, it has undergone a composite transformation.

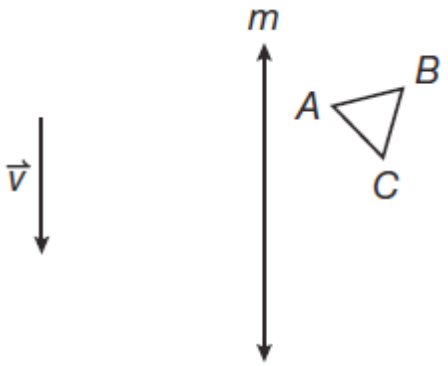
Composition of Two _____ - The composition of two isometries is an isometry.

Recall that translations, rotations, and reflections are all _____. A composite transformation with two or more of these transformations will also be an isometry.

A glide reflection is a composition of a translation and a reflection across a line parallel to the translation vector. Since a glide reflection combines translation and reflection, it is an isometry.

Example 1 Performing a Glide Reflection

Reflect ΔABC across line *m* and then translate it along $\vec{v}$.



SOLUTION

Math Reasoning

Verify In Example 1, what would be the result of performing the translation first, followed by the reflection?

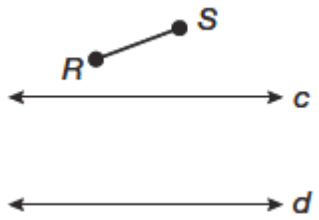
Two Reflections Across Two Parallel Lines - The composition of two reflections across two parallel lines is a translation.

If a figure is reflected twice, the end result is an image that represents a translation from the original preimage. Example 2 demonstrates this.

Example 2 Translating by Composite Reflection

Reflect the line segment across *c*, and then reflect the image across *d*. In the diagram, $c \parallel d$.

SOLUTION



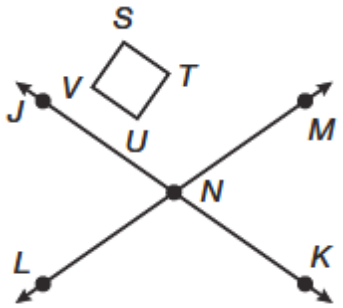
Two Reflections Across Two Intersecting Lines - The composition of two reflections across two intersecting lines is a rotation about the point of intersection. The angle of rotation is twice the angle formed by the intersecting lines.

If a figure is reflected across lines that intersect, it results in a rotation of the figure. Example 3 demonstrates this.

Example 3 Rotating by Composite Reflection

Reflect rectangle $STUV$ across $\overleftrightarrow{JK}$, and then reflect the image across $\overleftrightarrow{LM}$.

SOLUTION



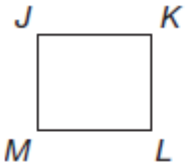
Math Reasoning

Analyze If you continue to reflect $STUV$ clockwise across $\overleftrightarrow{JK}$ and $\overleftrightarrow{LM}$, what will the final result be?

Applying the same transformations to a figure in a different order sometimes results in a different image.

Example 4 Analyzing Order of Composition

a. Perform transformations on the rectangle in the following order: reflection across a vertical line, rotation 90° clockwise, rotation 90° clockwise, and reflection across a horizontal line. What is the result?

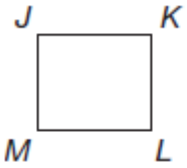


SOLUTION

Math Reasoning

Analyze Write one way to perform the composition in this example in a different order and get the same final image as the one in **a**.

b. Perform transformations on the original rectangle in part a in the following order: reflection across a vertical line, rotation 90° clockwise, reflection across a horizontal line, and rotation 90° clockwise. What is the result?



SOLUTION

c. Compare the final images in parts a and b. What do the results indicate?

SOLUTION

Exploration

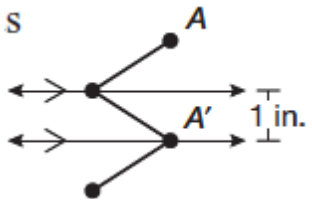
In this exploration, you will perform a composite transformation beginning with a given preimage. You will compare your final image to those of your classmates and to a final image provided by your teacher.

1. On grid paper, create ΔXYZ with $X(0,0)$, $Y(3,0)$, and $Z(0,4)$.
2. Follow the series of translations described by your teacher to generate an image based on composite transformations.
3. Redraw ΔXYZ on a new grid and follow a second series of translations described by your teacher to generate an image.
4. Compare your results for the two composite transformations with a classmate.
5. Check your results with the results that your teacher posts.

Example 5 Application: Design

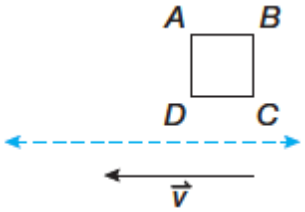
To create a design, Reina draws a line segment, and reflects it twice as shown. The reflection lines are parallel and are 1 inch apart. Find the distance from A to A'.

SOLUTION

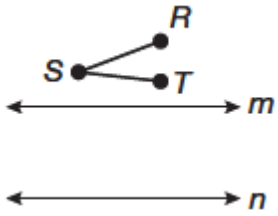


You Try!!!!!

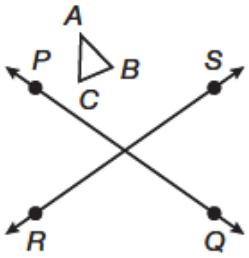
a. Reflect the square over the dashed line and translate it by $\vec{v}$.



b. Reflect the figure across *m*, and then reflect the image across *n*.



c. Reflect ΔABC across $\overleftrightarrow{PQ}$, and then reflect the image across $\overleftrightarrow{RS}$.



e. To create a new logo for a sweatshirt, a designer reflects the letter *T* across line *h*. This image is then reflected across line *j*. What single transformation could move the letter *T* from its starting point to its final position?