Lesson 91

Introduction to Trigonometric Identities

It has already been shown in previous lessons that $\tan\theta = \frac{\sin\theta}{\cos\theta}$. This is an example of a trigonometric identity. Trigonometric identities are expressions that relate any two trigonometric functions. Several trigonometric identities will be discussed later in this lesson, but two of the most commonly used trigonometric identities are two that relate sine and cosine.

Trigonometric Identities – The most commonly used trigonometric identities are below.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2 x + \cos^2 x = 1$$

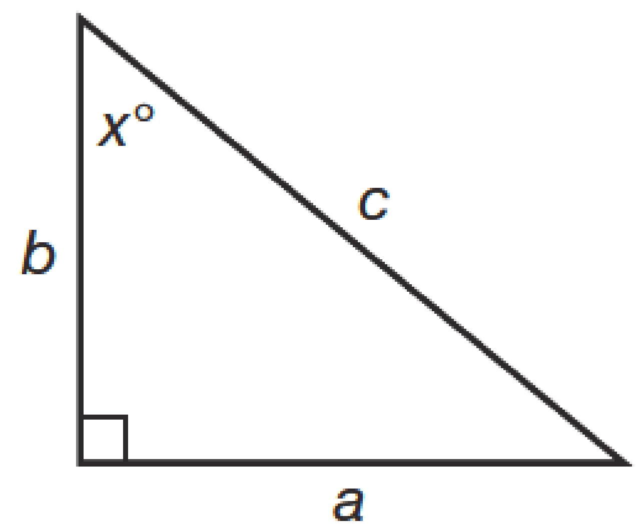
Reading Math

The square of a trigonometric function can be written two ways, as shown below.

$\cos^2 x = (\cos x)^2$

To prove the identity $\sin^2 x + \cos^2 x = 1$, substitute the trigonometric ratios for sine and cosine into the expression. The following proof uses the triangle shown.

$$\sin^2 x + \cos^2 x = 1$$
$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$$
$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$$
$$\left(\frac{a^2}{c^2} + \frac{b^2}{c^2}\right) \cdot c^2 = 1 \cdot c^2$$
$$a^2 + b^2 = c^2$$



This demonstrates that the identity given above is identical to the Pythagorean Theorem, which we already know is true.

Example 1 Relating Sine and Cosine

Find $\sin\theta$ if $\cos\theta = 0.5$.

SOLUTION

$\sin^2\theta + \cos^2\theta = 1$	Trigonometric Identity
$\sin^2\theta + (0.5)^2 = 1$	Substitute.
$\sin^2\theta + 0.25 = 1$	Simplify.
$\sin\theta \approx 0.87$	Solve.

By rearranging the trigonometric identities we already know, several more identities can be created.

Example 2 Building More Identities

a. Express $\tan\theta$ using only $\sin\theta$.

SOLUTION

Both of the identities are needed to complete this problem. First, rearrange the second identity to solve for cos θ.

$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - \sin^2\theta$$
$$\sqrt{\cos^2\theta} = \sqrt{1 - \sin^2\theta}$$
$$\cos\theta = \sqrt{1 - \sin^2\theta}$$

Now, substitute this into the tangent identity.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\tan\theta = \frac{\sin\theta}{\sqrt{1 - \sin^2\theta}}$$

Math Reasoning

Verify Confirm that the identity given in part **a** is valid by substituting an angle measure for θ and verifying the identity.

Example 2 Building More Identities

b. Express tan θ using only cos θ.

SOLUTION

Solving the same identity for cos θ gives a similar expression.

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

Substitute this into the tangent identity.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

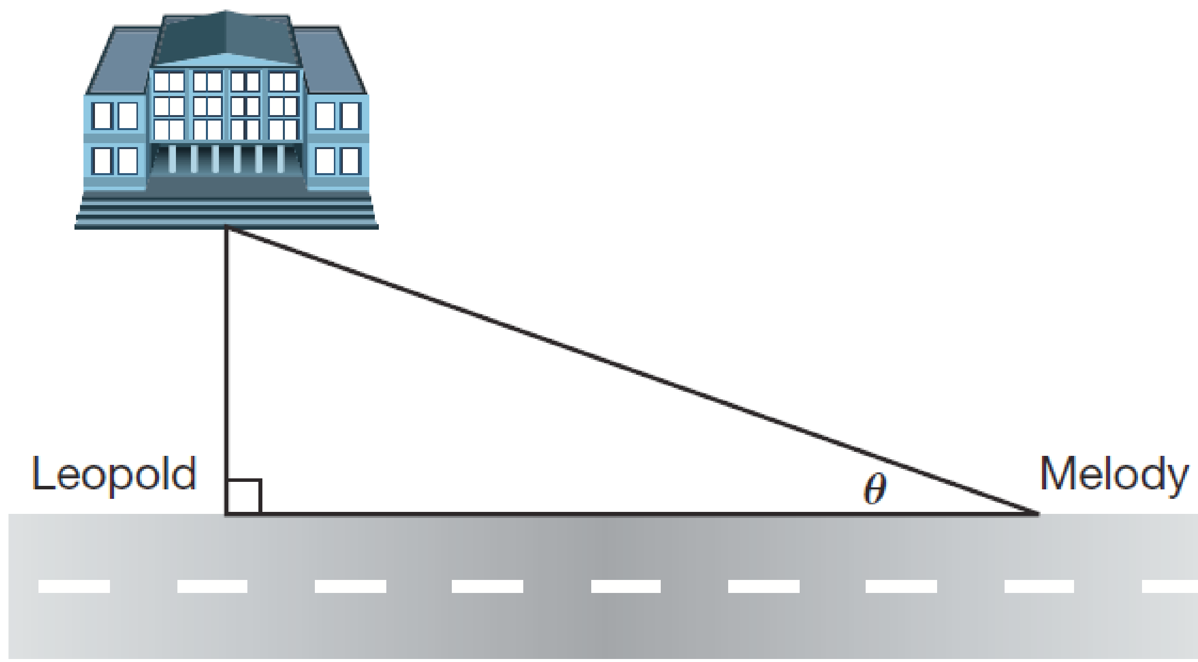
$$\tan\theta = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$$

This table outlines the relationships between the trigonometric functions.

Function	In terms of sine	In terms of cosine	In terms of tangent
$\sin\theta =$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}}$
$\cos\theta =$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\dfrac{1}{\sqrt{1+\tan^2\theta}}$
$\tan\theta =$	$\dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\dfrac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$

Example 3 Application: Estimating Distance

Leopold and Melody are standing on the street near their school, as shown in the diagram. Melody knows she is three times as far from the school as Leopold. What is the approximate ratio of Melody's distance from Leopold to Melody's distance from the school?



Example 3 Application: Estimating Distance

SOLUTION

If Melody is three times as far from the school as Leopold is, this implies that the ratio of the opposite leg to the hypotenuse is $\frac{1}{3}$.

So: $\sin\theta = \frac{1}{3}$

The problem asks for the ratio of the adjacent leg to the hypotenuse, which is a cosine function.

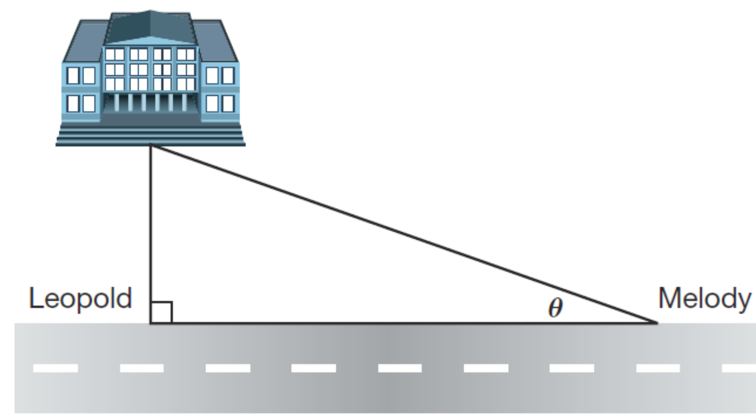
Use the identity to solve for cosine.

$$\sin^2 x + \cos^2 x = 1$$

$$\left(\frac{1}{3}\right)^2 + \cos^2 x = 1$$

$$\cos^2 x = \frac{8}{9}$$

$$\cos x = \frac{2\sqrt{2}}{3}$$



So the distance between Leopold and Melody is approximately $\frac{2\sqrt{2}}{3}$ times as far as the distance from Melody to the school.

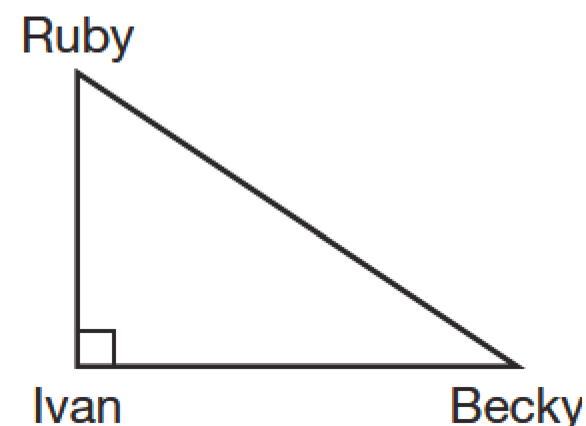
You Try!!!!

a. If $\sin^2\theta = 0.67$, what is the value of $\cos\theta$ to the nearest hundredth?

b. Express $\cos\theta$ in terms of $\tan\theta$ and show each step.

You Try!!!!

c. As shown in the diagram, Ruby is about twice as far from Becky as she is from Ivan. What is the approximate ratio of Ivan's distance from Becky to Ruby's distance from Becky?



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)