Geometry Lesson 91

Objective: TSW be introduced to trigonometric identities.

It has already been shown in previous lessons that $\tan\theta = \frac{\sin\theta}{\cos\theta}$. This is an example of a trigonometric identity. Trigonometric identities are expressions that relate any two trigonometric functions. Several trigonometric identities will be discussed later in this lesson, but two of the most commonly used trigonometric identities are two that relate sine and cosine.

Trigonometric Identities - The most commonly used trigonometric identities are below.
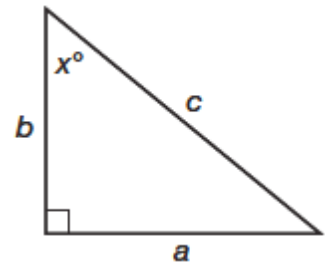
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
_____

> **Reading Math**
>
> The square of a trigonometric function can be written two ways, as shown below.
>
> $\cos^2 x = (\cos x)^2$

To prove the identity $\sin^2 x + \cos^2 x = 1$, substitute the trigonometric ratios for sine and cosine into the expression. The following proof uses the triangle shown.

$$\sin^2 x + \cos^2 x = 1$$

This demonstrates that the identity given above is identical to the Pythagorean Theorem, which we already know is true.

Example 1 Relating Sine and Cosine
Find $\sin\theta$ if $\cos\theta = 0.5$.
    SOLUTION

By rearranging the trigonometric identities we already know, several more identities can be created.

Example 2 Building More Identities
a. Express $\tan\theta$ using only $\sin\theta$.
    SOLUTION

> **Math Reasoning**
>
> **Verify** Confirm that the identity given in part a is valid by substituting an angle measure for θ and verifying the identity.

b. Express tan ϑ using only cos ϑ.

   SOLUTION

This table outlines the relationships between the trigonometric functions.

Function	In terms of sine	In terms of cosine	In terms of tangent
$\sin \theta =$	$\sin \theta$	$\sqrt{1 - \cos^2\theta}$	$\dfrac{\tan \theta}{\sqrt{1 + \tan^2\theta}}$
$\cos \theta =$	$\sqrt{1 - \sin^2\theta}$	$\cos \theta$	$\dfrac{1}{\sqrt{1 + \tan^2\theta}}$
$\tan \theta =$	$\dfrac{\sin \theta}{\sqrt{1 - \sin^2\theta}}$	$\dfrac{\sqrt{1 - \cos^2\theta}}{\cos \theta}$	$\tan \theta$

Example 3 Application: Estimating Distance
Leopold and Melody are standing on the street near their school, as shown in the diagram. Melody knows she is three times as far from the school as Leopold. What is the approximate ratio of Melody's distance from Leopold to Melody's distance from the school?

   SOLUTION
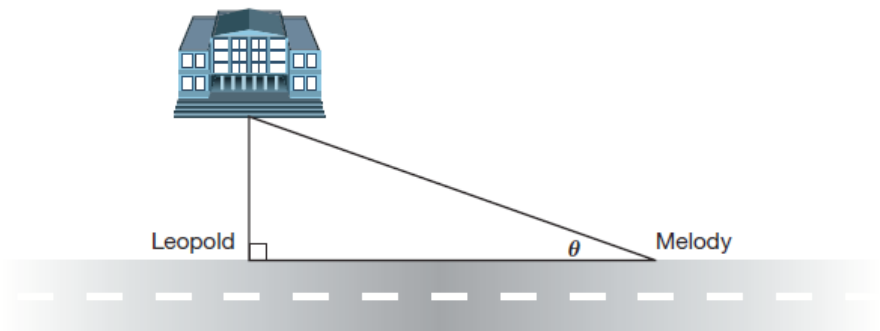
You Try!!!!

a. If $\sin^2 \theta = 0.67$, what is the value of $\cos \theta$ to the nearest hundredth?

b. Express $\cos \theta$ in terms of $\tan \theta$ and show each step.

c. As shown in the diagram, Ruby is about twice as far from Becky as she is from Ivan. What is the approximate ratio of Ivan's distance from Becky to Ruby's distance from Becky?