

Lesson 9

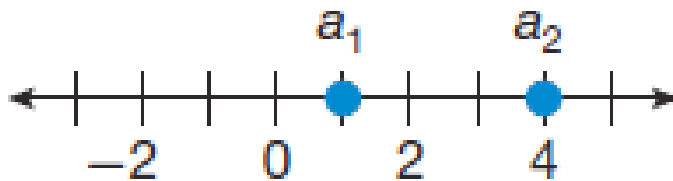
Finding Length: Distance Formula

Often, the length of a segment can be measured using a ruler. At other times, it may be necessary to find length by looking at a number line or a coordinate plane.

To find the distance between two points on a number line, take the absolute value of the difference between those points' coordinates.

$$d = |a_2 - a_1|$$

The distance between points a_1 and a_2 is $|4 - 1| = 3$.



Hint

An absolute value is used to determine distance and length because it is impossible for something to have a negative length, or for the distance between two points to be negative.

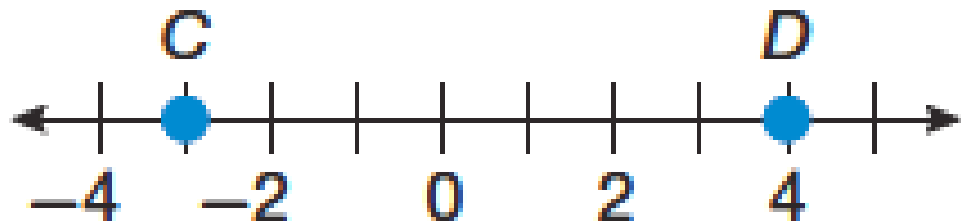
Example 1 Distance Between Two Points on a Line

Find the distance between the points on the number line.

SOLUTION

Use the formula:

$$\begin{aligned}d &= |a_2 - a_1| \\|4 - (-3)| &= |7| \\&= 7\end{aligned}$$



On a coordinate plane, the distance between two points can be found using the distance formula.

Distance Formula – In a coordinate plane, the distance d between two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 2 Using the Distance Formula

Find the distance between the two points.

SOLUTION

First, choose one point's coordinates to be (x_1, y_1) . The other point will be (x_2, y_2) . It does not matter which point is chosen. Let $(1, 2)$ be (x_1, y_1) and $(4, 6)$ be (x_2, y_2) .

Substitute into the distance formula.

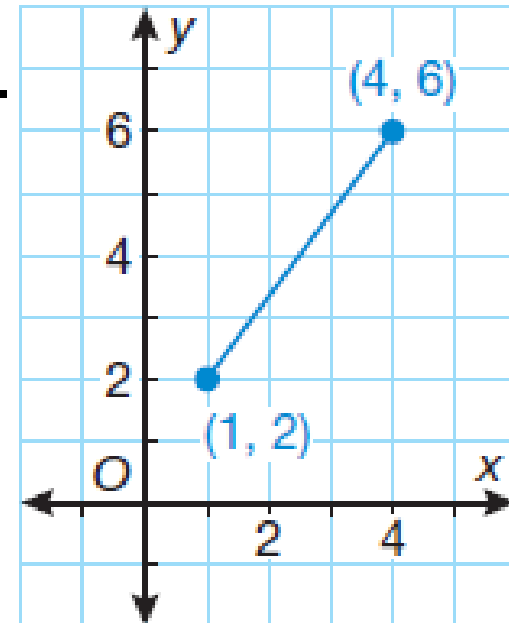
$$d = \sqrt{(4 - 1)^2 + (6 - 2)^2}$$

$$d = \sqrt{3^2 + 4^2}$$

$$d = \sqrt{9 + 16}$$

$$d = \sqrt{25}$$

$$d = 5$$



It does not matter which ordered pair is chosen to be (x_1, y_1) . It is important, however, that x_1 and y_1 come from the same ordered pair.

When two points share the same x -value or y -value, the distance formula can be simplified as shown in the next example.

Example 3 Distance Between Points That Share One Coordinate

Find the distance between the two points.

SOLUTION

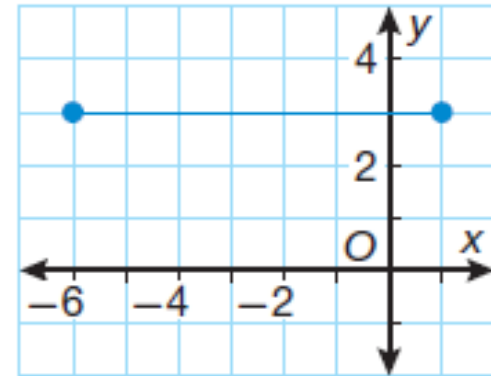
Since $y_1 = y_2$, we can substitute and simplify.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_2)^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + 0^2}$$

$$d = \sqrt{(x_2 - x_1)^2}$$



The square root and the square cancel, so with the two identical y -values, the distance formula becomes:

$$d = |x_2 - x_1|$$

Example 3 Distance Between Points That Share One Coordinate

An absolute value is used because squaring and then taking the square root of a number always results in a positive number. The resulting formula is identical to the one used to find distance on a number line.

Math Reasoning

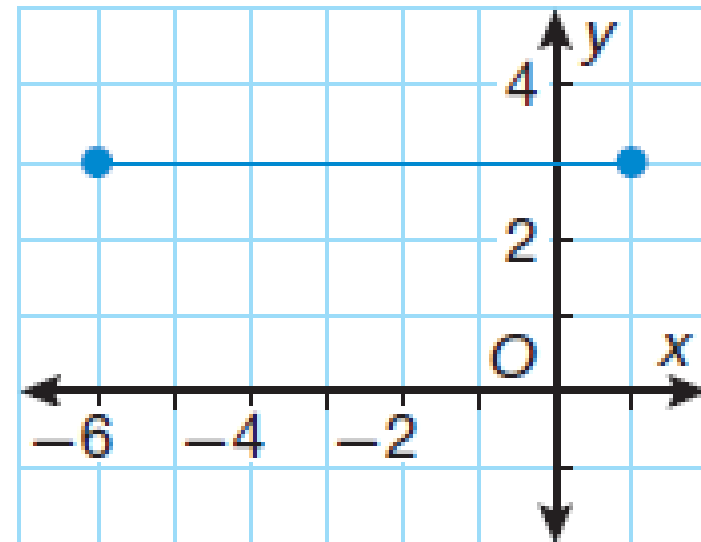
Verify Since the diagram shows a horizontal line, an easy way to verify the result of the distance formula is to simply count the number of unit squares the line crosses. Would the result be any different if you flipped the x and y coordinates of these points?

$$d = |x_2 - x_1|$$

$$d = |-6 - 1|$$

$$d = |-7|$$

$$d = 7$$



Example 4 Application: Navigation

Use the following map for each question.

The distance is measured from the dot on each building.

a. What is the distance from John's house to the school if each unit on the coordinate plane represents 100 meters?

SOLUTION

John's house is at $(4, 9)$ and the school is at $(2, 5)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

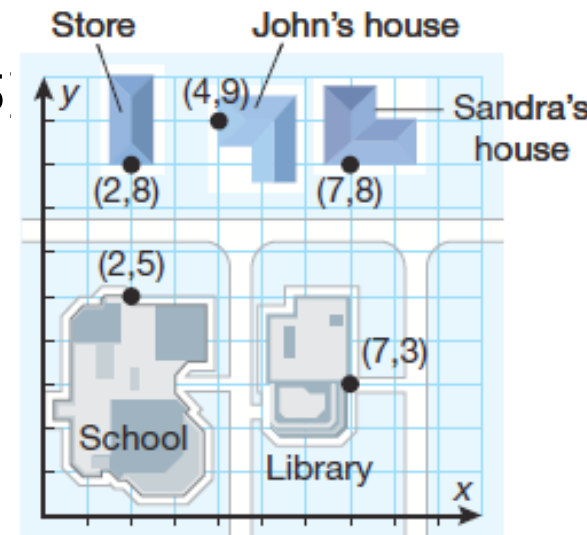
$$d = \sqrt{(4 - 2)^2 + (9 - 5)^2}$$

$$d = \sqrt{2^2 + 4^2}$$

$$d = \sqrt{4 + 16}$$

$$d = \sqrt{20}$$

$$d \approx 4.4721$$



Since each unit represents 100 meters, multiply the answer by 100. The distance from John's house to the school is about 447.21 meters.

Example 4 Application: Navigation

Use the following map for each question.

The distance is measured from the dot on each building.

b. What is the distance from Sandra's house to the store?

SOLUTION

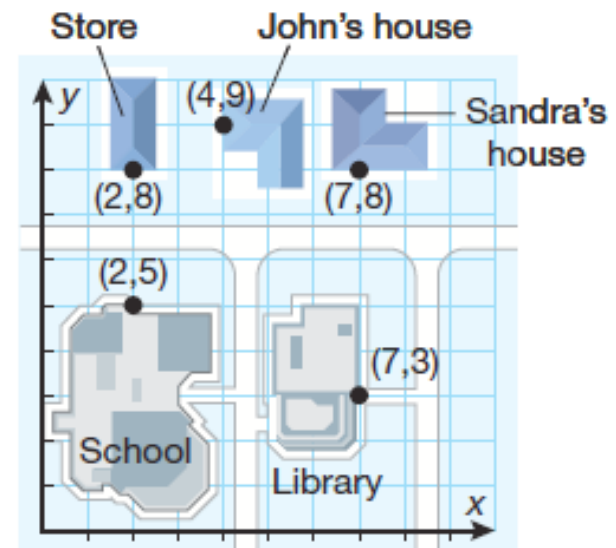
Sandra's house and the store have the same y -coordinate, so the formula for distance on a number line can be used.

$$d = |x_2 - x_1|$$

$$d = |7 - 2|$$

$$d = 5$$

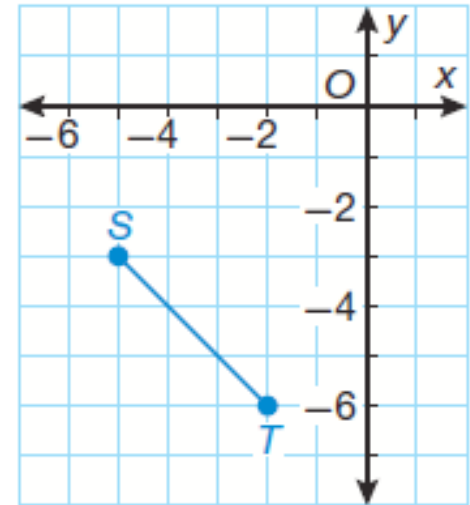
Since each unit represents 100 meters, the distance from Sandra's house to the store is 500 m.



You Try!!!!

b. What is the distance between points S and T ?
Round to the nearest hundredth.

4.24



c. Find the distance between the points $(2, 3)$
and $(2, -4)$.

7

Assignment

Page 55

Lesson Practice a–d (Ask Mr. Heintz)

Page 56

Practice 1–30 (Do the starred ones first)